Bounded Rationality

Henam Singla *

May 2019

Abstract

Lucas Critique was valid but under some assumptions.These days a lot of developments has occurred in the modern macroeconomics.My paper has also been inspired through one of the new upcoming terms.I examine an extremely simple and comprehend model of macroeconomics world, when the assumptions of fully rational expectations are relaxed. In this paper, I show that small deviations from Rational Expectations can lead to shocking results. Classical policy ineffectiveness proposition may not hold. It produces equilibria with policy effectiveness, output persistence and multiplier effects.The main thrust of this paper is to introduce the concept of Bounded Rationality in the model of overlapping wage contracts, where people in two-time periods are expecting in next period. PIP won't hold and output will show multiplier effects to monetary shock. Some reasons are mentioned for the failure of PIP. It shows that REH doesn't imply PIP to hold.

Keywords : Rational Expectation Hypothesis, Bounded Rationality, Overlapping Wage Contracts, Policy Ineffectiveness Proposition, Nominal Wage contracts, Macroeconomic effects

*Mentor-Dr. Divya Tuteja(IIFT)

1 BACKGROUND

Today, in this modern society(as we claim it to be), every field has advanced a lot. Economics is an upcoming area of interest of every individual where each individual wants to research in. Macroeconomics is also a sub-field of it which studies about the economy as a whole. Many models are based on the assumption of Perfect Rationality. Recent work hs been done about questioning this assumption. Whether this assumption we made is even rational? Do we know about all the activities(present or past) due to which a decision will take place? Are we fully informed, perfectly logical, and geared toward maximum economic gain (profit). "Theories that incorporate constraints on the information-processing capacities of the actor may be called theories of bounded rationality[1]. Thus, Simon claims, agents have only bounded rationality and are forced to make decisions not by 'maximization', but rather by satisfycing [2], i.e. setting an aspiration level which, if achieved, they will be happy enough with, and if they don't, try to change either their aspiration level or their decision. Will introducing Bounded rationality in the Lucas Critique proposition still hold? Since, we all know we can't have all the information needed all the time, we always have to make some guesses about it. This, bounded Rationality is like an elephant in a room, just too big to ignore. Models with Bounded rationality have got huge success in explaining the real world behavior consistent with it. There are alternative preferences such as "Dual self" model of Fudenberg and Levine, Hyperbolic discounting (Thaler, Laibson and others), Recursive utility (Epstein and Zin) etc. Bounded Rationality is also a learning which is an agent-based model. It replaces full optimization with Simon's "satisfying" optimization. There are some rule of thumb agents present in the economy. There is a famous theory known as PROSPECT THEORY (Kahneman) which assumes boundedly rational people becuase prefrences depend on framing.There are other theories of Colinsk and Arthur which tries to model it with their own models and have found excellent results in this field.

Some questions which are addressed here are:
1.If people are as smart as rational agents in our model, then why do you need to be taught about how to solve models?
2. why do so many students fail to get perfect scores?
If Rational agents dominate the argument, Shouldn't irrational agents be driven into non-importance? If irrational agents exist in the market, then things like unemployment, GDP,etc. irrational agents should matter. Rationality basically means you're clear in your argument while irrationality means wilderness. The most related idea to this will be SUB-OPTIMIZATION. In this process, people don't have full resources or due to optimization being unduly costly or impossible to do optimization, then they opt for this means. Though some economists argue that inherited emotions, social norms, sensitivity may improve economic parameters in ways which are outside the scope of economic theory. The question is not whether people are unboundedly rational; of course they are not. The question is whether they act approximately as if boundlessly rational; they do.Not only this, it's too fascinating to study this term.

"Rationality is like a giant elephant in a room ,even if you close your eyes and touch at several points, still you can't tell what it is!"

Which is why, we need Bounded Rationality since we can't get to know about all parameters needed to make a decision. Bounded Rationality is also an elephant in the room which is just too big to ignore.

[1]Simon, 1972, p.162

[2]The term 'satisficing' appears in Simon, 1956. Later Simon (1957, p.205) says "The key to the simplification of the choice process... is the replacement of the goal of maximizing with the goal of satisficing, of finding a course of action that is 'good enough

2 LITERATURE REVIEW

Analysis of Bounded Rationality		
Mathematical Analysis (with a Model)	Theoretical Analysis	Mathematical Analysis
Bomfim and Diebold; 1996*	Conlisk; 1996	Anderlini and Canning; 2000
Baak; 1998	Opreana, Fabian b, Brumarc, Bărbatd ; 2011	Mark Salmon; 1970
Jhang, Da and Wang, 2007**	Evans and Honkapohja	Eusepi and Preston; 2011
Hommes; 2010	Foss; 2003	Hommes, Massaro,Weber; 2015
Elsadany; 2010**	Dean and Ortoleva; 2012**	Axel Leijonhufvud;1993
Cars H. Hommes; 2008	W. Brian Arthur; 1994	Thomas J. Sargent, David Fand and Stephen Goldfeld;1973
Simon,H.A.;1982	Adam, Klaus;2002	Mankiw NG;1985
Robert E. Lucas, Jr and Edward C. Prescott;1971	Akerlof, George and Yellen, Janet L.;1985	Sargent, Thomas;1971
Hansen, Lars and Sargent, Thomas;1980	Lucas, Robert E;1967	Fisher,Stanley;1977
Heijdra(book)	Esther-Mirjam Sent;1997	John F. Muth;1961

1. * denotes major contribution by this paper
2. ** denotes microeconomics analysis also

My paper is most inspired from the paper of Bomfim[7]. That paper includes the concept of Bounded Rationality in the different macroeconomic models of macroeconomics. They also focused on strategic complementarity to show the truthfulness of ineffective proposition if small deviations from rational expectations is there. Rule-of-thumb which was taken by them has been used in my paper too. The results of monetary shocks are given on output and prices. Some light has been given to price surprises in aggregate supply function for supply elasticities. Then, there is a paper on testing Bounded Rationality by Baak[5]. He tested the heterogeneous expectations wit ha linear model with some assumptions. He tested the same model in US beef market. Using Maximum likelihood estimator(MLE),he observed that some of

the cattle market participants are boundedly rational with lower mean squared errors. It shows that even if market conditions don't change much, then also dynamics can b altered due to fraction of propel forming expectation changes. A very strong result was given by Anderlini[3], that if structural stability is there in the market, then it'll imply robustness to Bounded Rationality. This is more related to experimental economics. According to their result, even a very small deviation from fully rational expectations will lead to large impact on output iff their experimental setup is closed to some value at which point the model becomes structurally unstable. Far away values from this will give results yielding from rational expectations. One of the papers on Lucas Critique is by Salmon[25].In this paper, the theoretical basis of Lucas Critique is being challenged on the viewpoint of Robust Decision theory.It says that there can also be a policy change by the government. If dynamic game persists between private and government , much of the results of Lucas Critique may not hold. The endogeneity between private and government hs been taken into consideration in this paper. They have got that Critique can fail if rationality is extended and economic agents employ robust decision rules as analyzed by them.

One of the book which inspired me to do OVERLAPPING WAGE CONTRACT model is Ben J. heijdra[6]. This book has one dedicated portion toward expectations where Rational, adaptive and perfect foresight, etc. all types of models are taken into account. It's text is simple and can be comprehended easily. The Overlapping wage contract model is explained in much detail with the mathematical analysis as required. The paper which included Microeconomics analysis for testing Bounded Rationality is of Zhang[32].It included duopoly model modeled by two linear difference equations. It results in the observation that if there is any change in the speed of adjustment of Bounded Rationality, it can change the stability of Nah Equilibrium too. (Low⇒ Stable Nah Equilibrium, High⇒ Unstable). The paper which surveys learning-to-forecast experiments(LtFEs) with the laboratory testings is done by Hommes[18]. According to him, no homogeneous expectation hypothesis model fits the data. Cobweb markets can yield stable results. So, heterogeneous expectation is the main crux which may explain aggregate behavior across different market settings. Evans[14] article tries to explain the meaning of rational expectations in detail used in economics,mathematics and econometrics. Alsadany[3] also uses delayed duopoly game with Bounded Rationality. His paper concluded showing that firms which use delayed bounded rationality have a higher chance of reaching a Nash Equilibrium point. It included more of mathematical simulations and phase portraits. Dean and his co-autor[4], tried to use the relationship between loss aversion and violations of expected utility, are highly significant (included laboratory experiments).

Eusepi[13] showed a connection between expectations and business cycle fluctuations.Even in case of technology shocks, self-fulfilling expectations are possible. Conlisk[9] has a very different perspective towards approaching Bounded Rationality. He compared it with Hamlet and Puck. He approached with a theoretical way including Psychology, evidence,methodology, scarcity and human cognition. Beyond all these ingredients, he gave a small mathematical model for Rationality which studies the impact of deliberation cost, incentives,experience, complexity and market discipline. **Muth's** [22] paper talks about price fluctuations in an isolated market and it's expectations about the serially correlated disturbances. He took cross-sectional differences in expectations since aggregate effect is negligible as long as the deviation from the rational forecast for an individual firm is not strongly correlated with those of the others. Then effect of inventory speculation with market adjustments was also considered and cobweb theorem was also studied. Muth[22] also wrote a paper on optimal properties of exponentially weighted forecasts including random walk and white noise error term. Thomas[26] wrote about the "Accelerationist" view of the Phillips Curve tested empirically on US data and it's validity is subjected to a adequate and maintained hypothesis. Prescott and Lucas[5], their paper wrote about the term series behavior of Investment, prices and output

[3]A.A. Elsadany,Dynamics of a delayed duopoly game with bounded rationality, Mathematical and Computer Modelling, volume=52, issue=9-10, pages=1479-1489,date= 2010-11,doi= https://doi.org/10.1016/j.mcm.2010.06.011,

[4]Dean, Mark and Ortoleva, Pietro and Halevy, Yoram and Houser, Dan and Mccabe, Kevin and Pascal, Blaise and Rustichini, Aldo and Sautmann, Anja and Wilson, Alistair and Vesterlund, Lise,Estimating the Relationship between Economic Preferences: A Testing Ground for Unified Theories of Behavior,2019,month= 05,pages=45

[5]Investment Under Uncertainty,Robert E. Lucas, Jr and Edward C.Prescott,1971-09,Econometrica,volume=39,pages=659-

in a competitive market set up with a random demand. It determines about the "cost-of-adjustment type investment theory decisions. It studies about the industry equilibrium with shifting demand and costs of varying output. It also studies about the role of securities prices and not the SHADOW ones in the investment function. Lucas[20] also wrote on optimal investment policy extending the model of Strotz and Eisner to the n-good case. There were non-linear costs involved in the model with fixed and variable inputs.

Hansen[17] gave a model for multiple variable, linear stochastic rational expectations model.He explained through three interrelated factor demand models (individual, aggregate firm's decision, depletion of natural resources). These tell about optimal results or dynamic equilibria. Goldfeld[31] gives a standard model to analyze the relationship between interest and inflation(by the standard adaptive mechanism). Proper empirical tests are given for the models of Rational expectations. The analysis will be mainly done of natural rate hypothesis carried by Lucas. Oprean and his co-authors[10] mainly targeted three areas: scope from engineering to any kind of education reachable through permanent endeavor,approaches to education through sustainable development (with Bounded Rationality) and to move on from "inter-" to "trans-" disciplinary approaches to permanent education. Fred[21] has a fancy topic with Old Wine in Irrelevant New Bottles? His article is mainly about Behavioral economics telling about the importance of "Law and Society". Behavioral economics, enunciated by psychologists and other non-economists, is more concerned with how individuals reason rather than the behavioral outcomes of individuals' choices.

Arthur[4] gave the inductive reasoning for Bounded Rationality.It qualifies as an adaptive complex system.Economists have long been uneasy with the assumption of perfect, deductive rationality in decision contexts that are complicated and potentially ill-defined. The level at which humans can apply perfect rationality is surprisingly modest. Axel[19] tried to argue out if rationality is mathematically computable or not.Whether computers can give out results related to Rationality or not. Due to externalities and other market driven factors, he is of firm belief that one must abandon this entire mode of theory construction and rethink the matter from Alchian's evolutionary perspective. Dunn[12] said that Bounded Rationality is not fundamental uncertainty. He compared two terms coined by Simon: Bounded Rationality and fundamental uncertainty.

Pfajfar[24] questioned in his paper about designing monetary policy when expectation formation is not perfectly rational. It says that there are some instrumental rules using actual inflation and not the predicted one will produce lower inflation variability and reduce expectational cycles.Not only this, not all individuals have the same forecasting rules.Deak and his co-authors[11] constructed and examined the monetary polices of a New Keynesians model with Bounded Rationality and heterogeneous agents and explored the impact of monetary policy on it. In this paper, new concept of Internally Rational(IR) was introduced. Agents are IR given aggregate states and prices. IR results in an NK model with more persistence and a smaller policy space for rule parameters that induce stability and determinacy.Also Kahneman results match with the real world data.Cars[8] uses demand-supply cobweb analysis. Two main stories of Bounded Rationality were discussed: Adaptive learning and evolutionary selection (using simple rules such as naive expectations). Sargent's[27] paper recalls the state of a macroeconomic in the late 1960s exploring the legacies of the equilibrium concept. Wallace[28] and Yellen[2] and sent[29] had theoretical papers. The former one analyzes the effects of alternate ways of conducting monetary policy in an ad-hoc macroeconomic model(long-run neutrality) and the latter one tried to make connections with Simon's program of bounded rationality and artificial intelligence.Simon and Sargent didn't mean the same thing of artificial intelligence. Adam[1], Mankiw[23] and fischer[15] gave theoretical models on adaptive learning, small menu costs and long terms contracts with Bounded Rationality respectively. Simon[30] has brought the insights of decision theory, organization theory.

CRITICISM:Foss[16] has really different say to thing that in real world, Bounded rationality is used "Thinly", that is, it's not much necessary to produce results of the main theories in economics. The main

681,url=https://EconPapers.repec.org/RePEc:mcb:jmoncb:v:3:y:1971:i:3:p:721-25,publisher=The Econometric Society,jstor-1909571

reasons for which he has said this si due to absence of distinct, positive program for Bounded Rationality. It's also not used in Industrial Organization.

3 METHODOLOGY

What Lucas argued was that workers can't be fooled again and again, higher inflation will ultimately fail to lead to lower unemployment.In reality, all kinds of chance occurrences play an important role in economics. In a macroeconomic context one could think of stochastic events such as fluctuation in the climate, natural disasters, shock to world trade, etc. In such a setting, forecasting is a lot more difficult. Muth(1961) formulated the rational expectations hypothesis(REH) to deal with situations in which stochastic elements play a role.

3.1 THE BASIC MODEL

The basic postulates of REH are :
1. information is scarce and the economic system doesn't waste it
2. the way in which the expectations are formed depends in a well-specified way on the structure of the system describing the economy.

1. Aggregate Demand- Fiscal Policy is assumed to be held constant and monetary policy will be the only policy variable affecting the demand for output.

$$Y_d^t = \beta_0 + \beta_1(m_t - P_t) + \beta_2 E_{t-1}(P_{t+1} - P_t) + v_t; \beta_1, \beta_2 > 0$$

Y_d^t= log of real output demanded
$m_t - P_t$= log of real money supply
$E_{t-1}(P_{t+1} - P_t)$=Expected inflation rate
v_t= random error
The equation shows that demand depends on the money supply, expectation about the change in prices and prices itself. v_t represents all stochastic elements that impinge on the demand curve.

2. Aggregate Supply-Based on Lucas' work and hence known as the Lucas supply equation. Output will deviate from full employment or capacity output only when actual prices differ from those that the public anticipates. In logs,

$$Y_s^t = \alpha_0 + \alpha_1(P_t - E_{t-1}P_t) + u_t$$

Y_t^s= log of real output supplied
α_0 corresponds to full employment output Y_t^p
u_t = random error
The above equation shows that there is a production lag: suppliers must decide on the production capacity before knowing exactly what will be the price at which they can sell their goods. They make this decision on the basis of all the information that is available to them. Information set, Ω_{t-1},

$$\Omega_{t-1} \equiv (P_{t-1}, P_{t-2}, ...; Q_{t-1}, Q_{t-2}, ...; \alpha_0, \alpha_1, \beta_0, \beta_1; v_t, u_t \sim N(0, \sigma^2)$$

The agents know about all of the prices and quantities up to and including period $(t-1)$(they don't forget about the relevant past information. Obviously, they don't know about the current period's relevant variables. The agents also know about the structure of the economy they're operating in. Although the actual realization of the stochastic error terms is not known for period t, but the probability distribution

Figure 1: Normal distribution of u_t

of the stochastic variable is known.

3. Money Supply Rule-A monetary rule utilized by the policy authorities like

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 Y_{t-1} + e_t, E(e_t|I_{t-1}) = 0$$

The money supply at time t is a function of the last period's level of money supply and output plus a random, unpredictable shock e_t, which neither the policy authorities nor the public can predict.

4. Rational Expectations Hypothesis (REH)-Price Expectations are determined within the model in light of future developments of the money supply.

$$P_e^t = E_t(P_t|I_{t-1})$$

P_e^t represents the expected price in period-t
I_{t-1} represents the information set available at time-t of time period-(t-1) required to form expectations about period-t

If we try to look at the Policy Ineffectiveness Proposition assuming perfect rationality, it asserts that only unpredictable money supply fluctuations can affect output or any other real variables in the economy. But, if there is any predictable shock of the money supply, it can affect output, employment or any real variables in the economy. Recent example can be seen in India, where demonetization was done on 8 November 2016. The GDP growth rate for Q1'17-18 dropped to 5.7% compared to 7.9% a year ago. [6] After demonetization in November 2016, the number fell to 405 million in January–April 2017. So there was fall of 1.5 million in number of people employed. The number of persons employed was 406.7 million in 2016-17 which fell by 0.1% to 406.2 million in 2017-18. So the employment had stagnated which resulted in employment rate decline.[7] So, if we try to analyse the above model, equating demand-supply equations, we get,

$$P_t = \frac{\beta_0 - \alpha_0 + \beta_1 m_t + \alpha_1 E_{t-1} P_t + \beta_2 E_{t-1}[P_{t+1} - P_t] + v_t - u_t}{\alpha_1 + \beta_1}$$

Given that $E_{t-1}v_t = E_{t-1}u_t = 0$ and $E_{t-1}E_{t-1}P_t = E_{t-1}P_t$ and $m_t - E_{t-1}m_t = e_t$, we get,

$$P_t - E_{t-1}P_t = \frac{\beta_1 e_t + v_t - u_t}{\alpha_1 + \beta_1}$$

Using aggregate supply equation, $Y_t = f(e_t, v_t, u_t)$

This output equation shows that PIP holds for it. Thus output fluctuates randomly around the full employment level, with fluctuations due to unanticipated movements in the money stock. The behavior of output is therefore independent of any predictable counter-cyclical policy by the monetary authorities. The following points should be highlighted:

- The expected money was determined via the money-supply rule. The public knows that in fact, the money stock will differ from what they expect by e_t, but at time t-1, their best guess of e_t is zero.

[6] www.wikipedia.com
[7] www.wikipedia.com

- Price surprise is determined by unanticipated effects-unanticipated changes in money supply and random error terms.
- The expected forecast error is zero, reflecting the fact that, on average, the rational public is correct in its forecast.

Proposition: In this standard model, we've rational expectations, this model gives us the result that monetary policy is ineffective and output will also not show any perturbation in it.

3.2 Bounded Rationality Model

We'll introduce Bounded Rationality by having two types of agents in the economy: Ones who have rational expectations and the other who have a simple " Rule of Thumb " to use a simple forecasting rule. The rule for Bounded Rationality :

$$P_t^e = (1-\zeta)E(P_t|I_{t-1}) + \zeta P_t^*, 0 \leq \zeta \leq 1$$

Here, ζ represents the proportion of population who forms their expectations according to a simple rule (agents who don't have rational expectations). P_t^* denotes an expectation of time-t price, formed at time $(t-1)$ by any **rule-of- thumb** method. We adopt a simple rule to form expectations here, $P_t^* = P_{t-1}$ This is also known as NAÏVE expectations. Thus, the modified equation now becomes:

$$P_t^e = (1-\zeta)E(P_t|I_{t-1}) + \zeta P_{t-1}, 0 \leq \zeta \leq 1$$

Now, new Aggregate supply rule will be:

$$Y_s^t = \alpha_0 + \alpha_1(P_t - P_t^e) + \mu_t$$

here, μ_t are zero-mean stochastic shocks, uncorrelated over space and time (error terms).

$$Y_t^s = \alpha_0 + \alpha_1(P_t - (1-\zeta)E_{t-1}P_t - \zeta P_t) + \mu_t$$

We were given,

$$Y_t^d = \beta_0 + \beta_1(m_t - P_t) + \beta_2 E_{t-1}(P_{t+1} - P_t) + v_t$$

Equating, $Y_t^d = Y_t^s$

$$\alpha_0 + \alpha_1(P_t - (1-\zeta)E_{t-1}P_t - \zeta P_t) + \mu_t = \beta_0 + \beta_1(m_t - P_t) + \beta_2 E_{t-1}(P_{t+1} - P_t) + v_t$$

$$\alpha_0 + \alpha_1 P_t - \alpha_1(1-\zeta)E_{t-1}P_t - \alpha_1\zeta P_t + \mu_t = \beta_0 + \beta_1 m_t - \beta_+ 1 P_t + \beta_2 E_{t-1}(P_{t+1} - P_t) + v_t$$

We can solve this equation for P_t, we get,

$$P_t = \frac{(\beta_0 - \alpha_0) + \beta_1 m_t + \alpha_1(1-\zeta)E_{t-1}P_t + \beta_2 E_{t-1}(P_{t+1} - P_t) + \alpha_1\zeta P_{t-1} + v_t - \mu_t}{\alpha_1 + \beta_1}$$

$$E_{t-1}P_t = \frac{(\beta_0 - \alpha_0) + \beta_1 E_{t-1}m_t + \alpha_1(1-\zeta)E_{t-1}E_{t-1}P_t + \beta_2 E_{t-1}E_{t-1}(P_{t+1} - P_t) + \alpha_1\zeta E_{t-1}P_{t-1}}{\alpha_1 + \beta_1}$$

We can observe here that $E_{t-1}E_{t-1}P_t = E_{t-1}P_t$ and $E_{t-1}P_{t-1} = P_{t-1}$

The first equation holds because forming expectations in $(t-1)^{th}$ about the expectations of t^{th}'s period prices formed in $(t-1)$ period, it'll be same as forming expectations in $(t-1)$ period t^{th}'s period prices, since in period $(t-1)$, we know about the expectations of $(t-1)$ period.

The second equation holds because expectations formed in $(t-1)$ period about the prices in the same period will be the same as the prices, expectations don't matter since we would be knowing exactly about the prices. There is no need of expectations in it.

From the above equation, we can collect all terms of

$$E_{t-1}P_t = \frac{(\beta_0 - \alpha_0) + \beta_1 E_{t-1} m_t + \beta_2 E_{t-1} P_{t+1} + \alpha_1 P_{t-1}}{\beta_1 + \beta_2 + \alpha_1 \zeta}$$

If we now try to find out,

$$P_t - E_{t-1}P_t = \frac{(\beta_0-\alpha_0)(\beta_2+\alpha_1\zeta-\alpha_1)+\beta_1\beta_2 m_t - \alpha_1\beta_1 E_{t-1}m_t + (\beta_2 E_{t-1}P_{t+1}+\alpha_1\zeta P_{t-1})(\beta_2+\alpha_1\zeta-\alpha_1)}{(\alpha_1+\beta_1)(\beta_1+\beta_2+\alpha_1\zeta)} + \frac{(\alpha_1(1-\zeta)-\beta_2)E_{t-1}P_t + v_t - \mu_t}{\alpha_1+\beta_1}$$

Using the money-supply rule, which states that:

$$m_t = \sum_{i=1}^{\infty} \mu_{1i} u_{t-i} + \sum_{i=1}^{\infty} \mu_{2i} v_{t-i}$$

$\Rightarrow m_t - E_{t-1}m_t = 0$ since agents know the money supply rule in period t once they have lagged information. There is no stochastic element in the policy rule. Which basically means that,

$$P_t - E_{t-1}P_t = f(m_t, P_t - 1, P_{t+1}, v_t, \mu_t)$$

$$\Rightarrow y_t = f(m_t, P_t - 1, P_{t+1}, v_t, \mu_t)$$

This can now be compared with the previous result where PIP was valid. But, as we can see now that PIP doesn't hold here. Now, the anticipated part of the money supply can also affect output. The predictable part will not only affect prices but also the output of the economy.
It is apparent that policy is effective, output displays persistence, and there are potential multiplier effects. Even small values of ζ can potentially lead to large deviations from classical results.
Price surprise is determined by unanticipated effects-unanticipated changes in money supply and random error terms. **Monetary policy will now be *effective* under this new rule when Bounded Rationality is introduced.**

We know that in case of Rational Expectation Hypothesis, the counter-example which is given to show that output can be affected in case of rational expectations is given by Overlapping Wage Contracts. We'll now study this model in the latter case only and will try to analyze the results which we'll ge

3.3 Overlapping Wage Contracts

Consider the case where nominal contracts are decided for 2 periods. Assume that nominal wages are set such that the expected real wage is consistent with full employment. Therefore, in period- t there are two nominal wage contracts- half of the workforce is on the wage contract agreed upon in period- $t-1$ (to run in periods t and $t+1$ and the other half has a contract formulated in period- $t-2$ (to run in periods $t-1$ and t).
The following two rules become:

$$\left.\begin{aligned} w_t(t-1) &= E_{t-1}P_t = {}_{t-1}P_t^e \\ &= (1-\zeta)E(P_t|I_{t-1}) + \zeta P_{t-1}, \quad 0 \le \zeta \le 1 \\ w_t(t-2) &= E_{t-2}P_t = {}_{t-2}P_t^e \\ &= (1-\zeta)E(P_t|I_{t-2}) + \zeta P_{t-2}, \quad 0 \le \zeta \le 1 \end{aligned}\right\} \begin{array}{l}\text{Difference in the} \\ \text{information set in} \\ \text{the 2 contracts}\end{array}$$

Here,

$$E(P_t|I_{t-i}) = E_{t-i}P_t, \forall i = 1, 2$$

(NOTE: We can take any proportion of the contracts and not the only 1:1 ratio. This can be generalized even by taking λ and $(1-\lambda)$ and find out the solution. we have generalized this case by taking $\lambda = \frac{1}{2}$. The firms are perfectly competitive-there is only one output price.
New AS curve for the two-period contract case is:

$$y_t = \underbrace{\frac{1}{2}[P_t - w_t(t-1) + u_t]}_{\text{Output of firms with workers on one-year old contract}} + \underbrace{\frac{1}{2}[P_t - w_t(t-2) + u_t]}_{\text{Output of firms with workers on two-year (expiring) contract}}$$

$$y_t^s = \frac{1}{2}(P_t - (1-\zeta)E_{t-1}P_t - \zeta P_{t-1}) + \frac{1}{2}(P_t - (1-\zeta)E_{t-2}P_t - \zeta P_{t-2}) + u_t$$

$\Rightarrow$ Supply curve has two different surprise terms, differing in the information set.
AD will be as it was before :

$$y_t^d = m_t - P_t + v_t$$

Money supply rule will also be the same as before as:

$$m_t = \sum_{i=1}^{\infty} \mu_{1i} u_{t-i} + \sum_{i=1}^{\infty} \mu_{2i} v_{t-i}$$

To solve for P_t, we'll have to equate AD and AS,

$$y_t^s = \frac{1}{2}(P_t - (1-\zeta)E_{t-1}P_t - \zeta P_{t-1}) + \frac{1}{2}(P_t - (1-\zeta)E_{t-2}P_t - \zeta P_{t-2}) + u_t = y_t^d = m_t - P_t + v_t$$
$$\frac{1}{2}(P_t - (1-\zeta)E_{t-1}P_t - \zeta P_{t-1}) + \frac{1}{2}(P_t - (1-\zeta)E_{t-2}P_t - \zeta P_{t-2}) + u_t = m_t - P_t + v_t$$

Solving for P_t will give:

$$P_t = \frac{1}{2}(m_t + v_t - u_t + (\frac{1-\zeta}{2})(E_{t-1}P_t + E_{t-2}P_t) + (\frac{\zeta}{2})(P_{t-1} + P_{t-2}))$$

(1)

Upon taking expectations conditional upon period- $(t-2)$ information on both sides,

$$E_{t-2}P_t = \frac{1}{2}(E_{t-2}m_t + \underbrace{E_{t-2}v_t}_{=0} - \underbrace{E_{t-2}u_t}_{=0} + (\frac{(1-\zeta)}{2})(\underbrace{E_{t-2}E_{t-1}P_t}_{=E_{t-2}} + \underbrace{E_{t-2}E_{t-2}P_t}_{=E_{t-2}}) + (\frac{\zeta}{2})(E_{t-2}P_{t-1} + E_{t-2}P_{t-2})$$

There are some points to be noted in the above equation:

- $E_{t-2}m_t = E_{t-2}u_t = 0$ Since, these are just random shocks, their expected value be 0 (on an average).
- $E_{t-2}E_{t-1}(\quad) \Rightarrow$ you cannot say anything about the time ahead and therefore, you cannot change your mind.
- $E_{t-2}E_{t-2}(\quad) \Rightarrow$ We know about the expectations in period $(t-2)$, so there is no need of an additional expectation, it won't change my earlier expectation.

$$E_{t-2}P_t = \frac{1}{2}(E_{t-2}m_t + (\frac{1-\zeta}{2})(E_{t-2}P_t + E_{t-2}P_t) + (\frac{\zeta}{2})(E_{t-2}P_{t-1} + P_{t-2}))$$

Solving for $E_{t-2}P_t$ gives,

$$E_{t-2}P_t = \frac{E_{t-2}m_t + (\frac{\zeta}{2})(E_{t-2}P_{t-1} + P_{t-2})}{(1+\zeta)}$$

(2)

Consider equation. (1),

$$P_t = \frac{1}{2}(m_t + v_t - u_t + (\frac{1-\zeta}{2})(E_{t-1}P_t + E_{t-2}P_t) + (\frac{\zeta}{2})(P_{t-1} + P_{t-2}))$$

Take expectations conditional upon period $(t-1)$information on both sides,

$$E_{t-1}P_t = \frac{1}{2}(E_{t-1}m_t + \underbrace{E_{t-1}v_t}_{=0} - \underbrace{E_{t-1}u_t}_{=0} + (\frac{(1-\zeta)}{2})(\underbrace{E_{t-1}E_{t-1}P_t}_{=E_{t-1}} + \underbrace{E_{t-1}E_{t-2}P_t}_{=E_{t-2}}) + (\frac{\zeta}{2})(E_{t-1}P_{t-1} + E_{t-1}P_{t-2})$$

There are some points to be noted in the above equation:

- $E_{t-1}m_t = E_{t-2}u_t = 0$ Since, these are just random shocks, their expected value be 0 (on an average).
- $E_{t-1}E_{t-2}(\quad) \Rightarrow$ you cannot say anything about the time ago and therefore, can't change our mind. So, the new expectations will be same as the previous one. No changes will be introduced.
- $E_{t-1}E_{t-1}(\quad) \Rightarrow$ We know about the expectations in period $(t-1)$,there is no need of an additional expectation, it won't change my earlier expectation.
- $E_{t-1}P_{t-1} \Rightarrow$ We are forming expectations in period-$(t-1)$ about the same period's prices which we'll know, so expected price will be same as the original price as in that year.

$$E_{t-1}P_t = \frac{1}{2}(E_{t-1}m_t + (\frac{(1-\zeta)}{2})(E_{t-1}P_t + E_{t-2}P_t) + (\frac{\zeta}{2})(P_{t-1} + P_{t-2})$$

Solving for $E_{t-1}P_t$ gives,

$$E_{t-1}P_t = (\frac{2}{(1+\zeta)})(\frac{1}{2}E_{t-1}m_t + (\frac{(1-\zeta)}{4})E_{t-2}P_t + (\frac{\zeta}{4})(P_{t-1} + P_{t-2}))$$

Using equation (2), i.e. Substitute the expression for $E_{t-2}P_t$ in the above equation of $E_{t-1}P_t$, we'll get,

$$E_{t-1}P_t(\frac{(1+\zeta)}{2}) = \frac{1}{2}E_{t-1}m_t + (\frac{(1-\zeta)}{4})(\frac{E_{t-2}m_t + (\frac{\zeta}{2})(E_{t-2}P_{t-1} + P_{t-2})}{(1+\zeta)}) + (\frac{\zeta}{4})(P_{t-1} + P_{t-2})$$

$$E_{t-1}P_t\frac{(1+\zeta)}{2} = \frac{1}{2}E_{t-1}m_t + \frac{(1-\zeta)}{2(1+\zeta)}E_{t-2}m_t + \frac{\zeta(1-\zeta)}{4(1+\zeta)}E_{t-2}P_{t-1} + \frac{\zeta}{4}P_{t-1} + \frac{\zeta}{2(1+\zeta)}P_{t-2}$$

t This gives us the final value of $E_{t-1}P_t$ as :

$$E_{t-1}P_t = \frac{E_{t-1}m_t + \frac{1-\zeta}{2\zeta}E_{t-2}m_t + \frac{\zeta(1-\zeta)}{2(1+\zeta)}E_{t-2}P_{t-1} + \frac{\zeta}{2}P_{t-1} + \frac{\zeta}{1+\zeta}P_{t-2}}{(1+\zeta)}$$

Substitute the value of $E_{t-1}P_t$ and $E_{t-2}P_t$ in the expression for P_t,

$$\$P_t = \frac{1}{2}\left(m_t + v_t - u_t + \frac{1-\zeta}{2}\,\frac{E_{t-1}m_t + \frac{1-\zeta}{2\zeta}E_{t-2}m_t + \frac{\zeta(1-\zeta)}{2(1+\zeta)}E_{t-2}P_{t-1} + \frac{\zeta}{2}P_{t-1} + \frac{\zeta}{1+\zeta}P_{t-2}}{1+\zeta}\right) + \frac{1-\zeta}{4}\,\frac{E_{t-2}m_t + \frac{\zeta}{2}(E_{t-2}P_{t-1} + P_{t-2})}{1+\zeta} + \frac{\zeta}{4}(P_{t-1} + P_{t-2})$$

Upon Solving this equation, we get,

$$P_t = \frac{1}{2}m_t + \frac{1}{2}v_t - \frac{1}{2}u_t + \frac{(1-\zeta)}{4(1+\zeta)}E_{t-1}m_t + \frac{(1-\zeta)(3-\zeta)}{4(1+\zeta)}E_{t-2}m_t + \zeta P_{t-1} + \frac{3\zeta(1-\zeta)}{4(1+\zeta)}E_{t-2}P_{t-1} + \frac{2\zeta}{(1+\zeta)}P_{t-2}$$

Substitute the value of P_t in the AD equation,

$$y_t = m_t - P_t + v_t$$

$$\Rightarrow y_t = f(P_{t-1}, m_t, m_{t-1}, v_t, u_t, P_{t-2})$$

Assuming $E_{t-1}m_t = m_t$, y_t will still be a function of same variables. Monetary surprise term$= m_t - E_{t-2}m_t$
From

$$m_t = \sum_{i=1}^{\infty}\mu_{1i}u_{t-i} + \sum_{i=1}^{\infty}\mu_{2i}v_{t-i}$$

Rewrite, $m_t = \mu_{11}u_{t-1} + \mu_{21}v_{t-1} + \sum_{i=2}^{\infty}\mu_{1i}u_{t-i} + \sum_{i=2}^{\infty}\mu_{2i}v_{t-i}$

$$E_{t-2}m_t = \mu_{11}E_{t-2}u_{t-1} + \mu_{21}E_{t-2}v_{t-1} + \sum_{i=2}^{\infty}\mu_{2i}u_{t-i} + \sum_{i=2}^{\infty}\mu_{2i}v_{t-i}$$

Since, $E_{t-2}u_{t-1} = 0$ and $E_{t-2}v_{t-1} = 0$,

$$m_t - E_{t-2}m_t = \mu_{11}u_{t-1} + \mu_{21}v_{t-1}$$

Substitute in equation for y_t,

$\Rightarrow y_t = f(\mu_{11}, \mu_{21})$, where μ_{11} and μ_{21} are policy parameters.

PROPOSITION: $\Rightarrow$ PIP doesn't hold here. $\Rightarrow$ Output can be affected by monetary policy even under Rational and Bounded Expectations.

Reasoning-Between the time the 2 year contract is drawn up and the last year of operation of that contract, there is time for monetary authority to react to new information on recent economic disturbances. Because of 2 period contract, half the workers have implicitly based their contract wage on old information.

There can be a number of reasons why PIP fails.

For example, private agents may not have rational expectations, or there may be nominal price stickiness. Here, anticipated monetary policy is also able to cause deviations of output from its natural level, it may also be possible that it may affect the natural rate itself. A theoretic explanation to this could be: Mundell-Tobin effect, saying that a higher monetary growth rate depresses the real interest rate, and this boosts capital accumulation and the natural level of output.

4 CONCLUSION

The concept of Bounded Rationality is new and an upcoming topic in today's era. This is especially a topic of behavioral and social sciences.Rational economic behavior in which individuals maximize their own self interest is only one of many possible types of behavior that arise from natural selection. This paper examined about what Lucas said was actually holding true even in the case of Bounded Rationality. Since we can't be rational at all times,so there always exist bound on it.This paper took the Bounded Rationality rule from the paper of Antulio N. Bomfim and Francis X. Diebold[7].They only analyzed the Classical, New-Classical and New keynesians model with the introduction of Bounded Rationality in it. I've introduced the concept of Bounded Rationality into an otherwise simple and stylized aggregative economic model. The goal is to illustrate starkly the non-robustness of the policy-ineffectiveness proposition and related classical propositions to potentially small violations of the rational expectations assumptions. It is majorly addressing the overlapping wage contracts with Bounded Rationality. It is my hope that, just as Sargent and Wallace(1975) used their model to make clear macroeconomic effects of the main thrust in economic theory of the 1970s(rational expectations), so too my results make clear the effects of Bounded Rationality in Overlapping Wage contract model[8].It basically introduces the nominal rigidity in the economy since people fix their wages according to some rule before-hand.
The model with sophisticated and rule-of-thumb agents may be thought of as an approximation to a model in which all agents are rational in the sense that their expectations are mathematical expectations, but conditional upon a much more restrictive information set.Moreover, it would have the advantage of addressing concerns of this particular macroeconomic issue which may lack the basics of micro-foundations and will be analyzing the rule-of-thumb here.
Moreover, we identified in this paper that only with the analysis of Aggregate Demand and Aggregate Supply analysis with a given money supply rule, we can find some excellent results of effectiveness proposition. Not only theoretical, this is also used for practical applications. It's more connected to real world application since people, in general, are not perfectly rational. There always exist some bound on it.Contract wages are negotiated by the unionized labor. This is called overlapping because not all workers negotiate at the same time (e.g. in my model, in an year, half are negotiating this year and rest in the next period). Wage and price setting appears to be staggered.In these cases, the wage rate is not allowed to vary much.Workers sign a contract that specifies a fixed wage rate for each period for which the contract will last,that is, the wage rate has to be the same in each period of the contract. Model generates persistence in output due to monetary shocks and can have a persistent trade off between inflation and output. As we know, price is inertial but inflation is a jump variable.
Monetary policy is still effective in the staggered wage contract model(REH or Bounded Rationality). The effectiveness of monetary policy doesn't require anyone to be fooled.So, there is no dispute in the conclusion that monetary policy can affect price level behavior. For the argument that price changes are costly, it is highly desirable to maintain price stability.This paper also examines the case if all contracts are in one period,

- If REH $\Rightarrow$ PIP is valid
- If Bounded Rationality $\Rightarrow$ PIP is not valid

An attempt by monetary authorities to exploit the structure of the contracts can lead to reopening of the same and can persuade to have new structure of the contracts. Then, in the next period, their policies, might not be, necessarily stabilizing.[9]

[8] **Taylor contract or staggered wage contract**.
[9] **Another popular staggered wage contract model is by Calvo(1983), includes Poisson process**.

References

[1] Adam, Klaus. *Adaptive Learning and Cyclical Behavior of Output and Inflation*. Macroeconomics. University Library of Munich, Germany, 2002. URL: https://EconPapers.repec.org/RePEc:wpa:wuwpma:0211013.

[2] Akerlof, George and Yellen, Janet L. "A Near-Rational Model of the Business Cycle, with Wage and Price Inertia". In: *The Quarterly Journal of Economics* 100.Supplement (1985), pp. 823–838. URL: https://EconPapers.repec.org/RePEc:oup:qjecon:v:100:y:1985:i:supplement:p:823-838..

[3] Anderlini, Luca and Canning, David. "Structural Stability Implies Robustness to Bounded Rationality". In: *Journal of Economic Theory* 101.2 (Dec. 2001), pp. 395–422. DOI: 10.1006/jeth.2000.2784.

[4] Arthur, W. Brian. "Inductive Reasoning and Bounded Rationality". In: *The American Economic Review* 84.2 (May 1994), pp. 406–411. American Economic Association: 2117868.

[5] Baak, Saang Joon. "Tests for bounded rationality with a linear dynamic model distorted by heterogeneous expectations". In: *Journal of Economic Dynamics & Control* 23.9-10 (Sept. 1999), pp. 1517–1543. DOI: 10.1016/S0165-1889(98)00082-7.

[6] Ben J. Heijdra, Frederick van der Ploeg. *The Foundations of Modern Macroeconomics*. Oxford University Press, 2002. ISBN: 0198776187, 9780198776185.

[7] Bomfim, Antúlio N. and Diebold, Francis X. "Bounded Rationality and Strategic Complementarity in a Macroeconomic Model: Policy Effects, Persistence and Multipliers". In: *The Economic Journal* 107.444 (Sept. 1997), pp. 1358–1374. JSTOR: 2957740.

[8] Cars, Hommes. "Bounded Rationality and Learning in Complex Markets". In: *Handbook of Research on Complexity* (Feb. 2007), p. 44.

[9] Conlisk, John. "Why Bounded Rationality?" In: *Journal of Economic Literature* 34.2 (June 1996), pp. 669–700. JSTOR: 2729218.

[10] Constantin Oprean Ralf D. Fabian, Cristina I. Brumar Boldur E. Bărbat. "Bounded Rationality for "Just In Time" Education". In: *Procedia - Social and Behavioral Sciences* 30 (2011), pp. 983–987. DOI: 10.1016/j.sbspro.2011.10.191.

[11] Deak, Szabolcs et al. "Internal rationality, learning and imperfect information". In: *Learning and Imperfect Information (December 12, 2017)* (2017). DOI: https://dx.doi.org/10.2139/ssrn.3091876.

[12] Dunn, Stephen P. "Bounded Rationality Is Not Fundamental Uncertainty: A Post Keynesian Perspective". In: *Journal of Post Keynesian Economics* 23.4 (2001), pp. 567–587. Taylor Francis, Ltd. 4538749.

[13] Eusepi, Stefano and Preston, Bruce. "Expectations, Learning, and Business Cycle Fluctuations". In: *The American Economic Review* 101.6 (Oct. 2011), pp. 2844–2872. JSTOR: 23045661.

[14] Evans, George W, Honkapohja, Seppo, et al. "Expectations, learning and monetary policy: an overview of recent research". In: *Monetary policy under uncertainty and learning* 13 (2009), pp. 27–76.

[15] Fischer, Stanley. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule". In: *Journal of Political Economy* 85.1 (1977), pp. 191–205. URL: https://EconPapers.repec.org/RePEc:ucp:jpolec:v:85:y:1977:i:1:p:191-205.

[16] Foss, Nicolai J. "Bounded rationality in the economics of organization: "Much cited and little used"". In: *Journal of Economic Psychology* 24.2 (Apr. 2003), pp. 245–264. DOI: 10.1016/S0167-4870(02)00206-4.

[17] Hansen, Lars and Sargent, Thomas. *Linear rational expectations models for dynamically interrelated variables*. Working Papers 135. Federal Reserve Bank of Minneapolis, 1980. URL: https://EconPapers.repec.org/RePEc:fip:fedmwp:135.

[18] Hommes, Cars. "The heterogeneous expectations hypothesis: Some evidence from the lab". In: *Journal of Economic Dynamics Control* 25 (Oct. 2010), pp. 1–24. DOI: 10.1016/j.jedc.2010.10.003.

[19] Leijonhufvud, Axel. "Towards a Not-Too-Rational Macroeconomics". In: *Southern Economic Journal,* 60.1 (July 1993), pp. 1–13. Southern Economic Association: 1059926.

[20] Lucas, Robert E. "Optimal investment policy and the flexible accelerator". In: *International economic review* 8.1 (1967-02), pp. 78–85.

[21] McChesney, Fred S. "Old Wine in Irrelevant New Bottles?" In: *Supreme Court Economic Review* 21.1 (Jan. 2014), pp. 43–76. JSTOR: 675265.

[22] Muth, John F. "Rational Expectations and the Theory of Price Movements". In: *Econometrica,* 29.3 (1961-07), pp. 315–335.

[23] NG, Mankiw. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly". In: *Quarterly Journal of Economics* (1985), pp. 529–537.

[24] Pfajfar, Damjan and Žakelj, Blaž. "Inflation expectations and monetary policy design: Evidence from the laboratory". In: *Macroeconomic Dynamics* 22.4 (2018), pp. 1035–1075. DOI: https://doi.org/10.1017/S1365100516000560.

[25] Salmon, Mark and Marcellino, Massimiliano. "Robust Decision theory and The Lucas Critique". In: *Cambridge University Press* 6(01) (Feb. 2002), pp. 167–185. DOI: 10.1017/S1365100502027086.

[26] Sargent, Thomas. "A Note on the 'Accelerationist' Controversy". In: *Journal of Money, Credit and Banking* 3.3 (1971), pp. 721–25. URL: https://EconPapers.repec.org/RePEc:mcb:jmoncb:v:3:y:1971:i:3:p:721-25.

[27] Sargent, Thomas J. "Expectations and the nonneutrality of Lucas". In: *Journal of Monetary Economics* 37.3 (1996), pp. 535–548.

[28] Sargent, Thomas J. and Wallace, Neil. ""Rational" Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule". In: *Journal of Political Economy* 83.2 (1975-04), pp. 241–254. JSTOR: 1830921.

[29] Sent, Esther-Mirjam. "Sargent versus Simon: bounded rationality unbound". In: *Cambridge Journal of Economics* 21.3 (1997-05), 323–338. DOI: https://doi.org/10.1093/oxfordjournals.cje.a013673.

[30] Simon, H.A. "Models of Bounded Rationality". In: *Open Journal of Philosophy* 4 (Nov. 2014).

[31] Thomas J. Sargent, David Fand and Goldfeld, Stephen. "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment". In: *Brookings Papers on Economic Activity,* 1932.2 (1973), pp. 429–480.

[32] Zhang, Jixiang, Da, Qingli, and Wang, Yanhua. "The dynamics of Bertrand model with bounded rationality". In: *Chaos, Solitons & Fractals* 39.5 (2009), pp. 2048–2055.