Coping with Frustration and Guilt-aversion

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Abstract

In this paper, I've tried to review the paper of frustration and anger in games. This paper defines a frustration function with three types of games: Simple Anger, Anger with Blaming Behavior and Anger with Blaming Intentions. All of these three models are studied in detail, with some new example. The notations the author have used are defined clearly and expllained in my own words. I've tried to extend the paper by introducing a guilt-aversion function and define a simple game on it. Guilt aversion is linked to frustration and a simple utility function is function thereon.

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1 Introduction

In this paper, I try to extend the paper named "Frustration and Anger in Games" of Pierpaolo Battigalli, Martin Dufwenberg and Alec Smith[12]. Not only this, I have reviewed the same paper. I have tried to give some proofs which were not given in the paper and tried to interpret the same in my own way.
Anger, as mentioned by them can take several forms such as:
a) When local football teams favored to win instead of lose, the police get more reports of husbands assaulting wives (Card & Dahl 2011)[28]. Do unexpected losses spur vented frustration?
b) In corporate world, subordinates tend to be scolded by their employers leading to loose in interest of employee and this eventually leads to inefficient results and hampering in the organization.
One of the studies investigates the stability of emotional influences on economic decision making. While standard economic theory has emphasized the rationality of economic agents, dual-system models of decision-making argue that human behavior can be viewed as the outcome of the interaction between a (fast) affective system that reacts to emotions and motivational drives and a (slower) goal-based cognitive system.1 Evidence indicates that the affective system tends to react first and to initially hold sway over the cognitive system.
However, the experimental comparisons between the two hypotheses have never been made in an environment where subjects have an opportunity for pre-play communication. As we clarify below, this is a serious omission because whether an opportunity for communication exists or not is likely to be one of the most important determinants of the feeling of guilt. To make this point clear, we consider the following new version of the guilt aversion hypothesis called personal guilt-aversion. According to it, people feel guilty when they betray another person's expectation with that expectation having been raised by their very own actions, typically by their promises.3 Unlike the original version, it is shown that personal guilt aversion is consistent with all extant experimental results in the literature introduced above. Note that these results have hitherto not been explained by any single model, including the original guilt aversion hypothesis. In this sense, it is justifiable to adopt personal guilt aversion as a model of guilt-aversion. In light of personal guilt-aversion, whether an opportunity for communication exists or not is a key determinant of the feeling of guilt. Particularly, the existence of an opportunity for communication (or, more generally, an action that can raise another person's expectation) is a prerequisite for the feeling of guilt. However, in the literature, the guilt aversion hypothesis and alternative hypotheses have been experimentally compared only under settings where subjects have no opportunity for such communication. As a result, the validity of personal guilt aversion cannot be tested by any extant experiment. Motivated by this observation, we design and conduct a new experiment that can test the personal as well as original versions of the guilt aversion hypothesis. To examine these questions, we study behavior in a particularly one—sided and unfair bargaining environment, the well-known ultimatum game (Güth, Schmittberger, and Schwarze, 1982; Nowak, Page, and Sigmund, 2000; and many others)[28][37]. Abundant experimental evidence documents that in the ultimatum game unfair offers from proposers are frequently rejected by responders, even though responders forgo money by doing so. This suggests that rejection rates in the

ultimatum game might fall if a cooling-off period is imposed. Traditional game theory is not a rich enough toolbox to adequately describe many psychological or social aspects of motivation and behavior. The traditional approach assumes utilities depend only on which actions are chosen, but if decision makers are emotional or care for the intentions, opinions, or emotions of others utilities may depend also on which beliefs (about choices, beliefs, or information) players harbor. These are just a few cases where frustration and anger leading to negative economic outcomes. This sometimes also lead to players getting frustrated at a level where they want to exit the game even. In this paper they have introduced a Frustration function in their own manner. But, as pointed out in the paper itself, there can be other ways to define frustration and then, analyzing it in the game form. If you accept a 50-50 gamble between 0 and 2,000, there is a 50% chance that you will be disappointed when the lottery is resolved. You may prefer to swap the lottery ticket for a sure 950 not so much because of arguments about decreasing marginal value, but because doing so removes the possibility of disappointment[54]. Of course, someone who feels that the "thrill of victory" is worth the possible "agony of defeat" may take the opposite choice. There are many other "reference effect" phenomena. A bonus of 5,000 may exceed one's expectations, but still lead to dissatisfaction if you learn that your colleague got a bonus of 10,000. Perhaps the most influential reference point is the status quo of the decision maker. It has been widely observed that a decision maker will make significant economic trade-offs to remove the possibility of a net loss on a transaction. Building a utility model that incorporates all of these effects may well be desirable not only to provide a better description of behavior, but also, to the extent that a decision maker is prepared to trade off dollars explicitly to gain a state of psychological satisfaction, for prescriptive purposes.
The intellectual home for our exercise is what has been called psychological game theory. This framework—originally developed by John Geanakoplos, David Pearce, and Ennio Stacchetti (1989)[48] and recently extended by Battigalli and Dufwenberg (2005)[11] (henceforth B&D)—allows players' utilities to depend on beliefs (about choices, states of nature, Guilt in Games By Pierpaolo Battigalli and Martin Dufwenberg[13] or others' beliefs) as is typical of many emotions. Our approach formalizes Baumeister, Stillwell, and Heatherton's (1994)[2] remark that guilt depends on a failure "to live up to [others'] expectations," and embraces some previous related theoretical and experimental results on "trust games." We refer to, e.g., Gary Charness and Dufwenberg (2006)[49] for elucidation on the role of guilt in that specific context, which space constraints prevent us from repeating here as we develop a theory for general games.

2 Literature Review

They have made some basic assumptions where in each game introduced, the player at the root level (first mover) is not frustrated. So, he assumes the game to be a normal form game. But, at the subsequent stages, if the game doesn't fold according to what the other player expects, then anger may arise. The other player, in the latter scenario may want to punish the first mover according to the sensitivity of his anger. Marcus-Newhall et al. (2000)[53]

introduced displaced aggression in Simple Anger games. But, now they are also introducing Anger from Blaming Behaviour (ABB) and Anger from Blaming Intentions (ABI).
In pure-strategy sequential equilibria, frustration will arise only off the equilibrium path because everything is known in that form of game. On the equilibrium path, the player is satisfied and the player knows what the other player will move given his action and the other player also knows what he will move given his own action and this process goes on. So, it is a common knowledge.
Recently, however, there has been some resurgence of interest in the affective attitude formation and behavioral response. One model of the affective basis aggression has been developed by Spector and colleagues (Chen and Spector, 1975, 1978, 1997;Storms and Spector, 1987)[42][37]. This model has its roots in the Miller frustration-aggression theory (Dollard, Doob, Miller, Mowrer [65] and focuses on the interplay of affective and behavioral responses to certain types of The Dollard-Miller [10] model views aggression as a consequence of frustration. occurs when an instigated goal-response (or predicted behavioral sequence) is interdicted. It is possible that the individual may find a substitute response for the response; however, if that does not occur, the individual may respond with some covert, externally or internally directed) of aggression (Dollard et al., 1939)[13]. aggressive response takes will be strongly influenced by the individual's perception likelihood of being punished. According to Dollard et al. (1939)[13] 'the inhibition aggression varies directly with the strength of the punishment anticipated for the that act' (p. 37). Thus overt expressions of aggression through counterproductive behavior would be expected to be related to the perception that one could 'get without being caught or punished. Subsequent research on this frustration-aggression sequence highlights the emotional reaction. In particular, Berkowitz [8] critiqued the original frustration-aggression thesis for neglecting the mediating role of the arousal of negative affect on between frustrations and fight-or-flight behavior (Spielberger, Reheiser and Sydeman,[71] two important aspects of this reaction are that the emotion is aversive, and that increased physiological arousal (Spector, 1978)[73]. Another criticism of Dollard et approach was too mechanistic, ignoring cognitive and dispositional processes. Comprehensive model would consider the effects of belief-based variables such as locus of as personality dispositions.
Abundant evidence documents (Let me sleep on it: Delay reduces rejection rates in ultimatum games Veronika Grimm a, Friederike Mengel)[43] that people are willing to "burn money" in order to punish unfair behavior by others. This is commonly attributed to negative emotions such as anger, blame, disgust, or resentment. There is, however, a common implicit understanding that delaying a reaction may mitigate those negative emotions and hence lead to more moderate reactions. This is often alluded to when people say "Let me sleep on it before I make a decision". Public administrations, for example, make use of this by communicating bad news on Friday afternoons such that people cannot react until Monday morning.[74] In this article we show that "having a break" can be enough. Delaying decisions by only 10 min after receiving bad news drastically changes human behavior in a controlled laboratory environment. They study the effect of delaying decisions in the context of the so called Ultimatum Game. In this game a proposer proposes a division of say 10 Euros to a responder. In this study, [64] we show that the mere fact of delaying the acceptance decision by approximately 10 min drastically and significantly changes human behavior in the Ultimatum Game. Acceptance rates for low offers increase from 0 to

15% without delay to around 60–80% with delay. Delay is implemented by letting participants answer a questionnaire immediately after receiving the proposal. Earlier studies have shown that delay does not have a significant effect on acceptance rates if participants have already expressed their negative emotions either explicitly or implicitly (via a decision they are asked to revise later). We confirm this in a control treatment where responders make an immediate decision which they can reverse after having answered the questionnaire. In our original design, however, where participants are not asked for any reaction immediately after receiving the proposal, delay has a dramatic effect. Our findings suggest that humans manage to cognitively control negative emotions[37]. If the responder accepts the proposal, both parties receive the suggested amounts. If he rejects both receive nothing. This game has been widely studied to investigate economic decision making and one of the most robust findings in experimental economics is that low offers (of say 1 or 2 Euros) are almost always rejected. Those empirical observations have triggered the development of a literature questioning the standard model of economic decision making which predicts that responders should accept any amount offered and consequently, proposers should never offer more than the minimal amount possible. By now a large body of work exists that incorporates social motives into the description of preferences in order to accommodate the observed behavior. In earlier work (Bell [1982, 1983][5], also Loomes and Sugden [1982])[56], I have explored the implications of regret as a factor in risk attitude. Regret is a psychological reaction to making a wrong decision, where wrong is determined on the basis of actual outcomes rather than on the information available at the time of the decision. Just as disappointment is caused by comparing an outcome with prior expectations, so regret is caused by comparing an outcome with the payoff one could have had by making a different choice. For example, if you are given a 50-50 lottery between 0 and 10 and lose, you will suffer disappointment. Had you selected the same lottery over an alternative of 4 for sure and lost, you would have suffered both disappointment and regret. For one to suffer only regret and not disappointment, the outcome of a chosen lottery would have to be exactly equal to one's expectations, but less than one could have obtained (ex post) from an alternative lottery. There are many other "reference effect" phenomena.[63] A bonus of 5,000 may exceed one's expectations, but still lead to dissatisfaction if you learn that your colleague got a bonus of 10,000. Perhaps the most influential reference point is the status quo of the decision maker. It has been widely observed that a decision maker will make significant economic trade-offs to remove the possibility of a net loss on a transaction. Building a utility model that incorporates all of these effects may well be desirable not only to provide a better description of behavior, but also, to the extent that a decision maker is prepared to trade off dollars explicitly to gain a state of psychological satisfaction, for prescriptive purposes.

In the work of Sell, Cosmides, and Tooby (2009)[34], they argue that anger is the result of natural selection for behaviors that resolve bargaining conflicts in favor of the anger-prone individuals. The way authors have developed this paper is that they have tried to define anger in such a way that if first mover deviates from the material-payoff equilibrium for both the players, then the leader gets more payoff but in that case, the follower (anger-prone individual) would end up punishing the leader and giving him less payoff than he would have got in any other case.

In a commonly used anecdote to illustrate displaced aggression, a man is berated by his boss but does not retaliate because he fears losing his job. Hours later, when he arrives home to the greeting barks of his dog he responds by kicking it. Conceptually, displaced aggression can be defined as a level of aggression toward a target that, in terms of the tit-for-tat rule (Axelrod, 1984)[65], incommensurately exceeds that which is ordinarily seen as justified by the level of provocation emitted by that target. In exceeding the aggression warranted by the target's behavior, it reflects the failure to respond aggressively toward the source of a temporally antecedent provocation, or in this case the berating boss. The notion that frustration leads to aggression is commonly known as the frustration-aggression hypothesis (Dollard, Doob, Miller, Mowrer, & Sears, 1939)[17][19]. Several conditions influence the intensity and/or frequency of aggression: (a) greater levels of frustration, (b) stronger expectations of reaching a goal, and/or (c) increased interference with goal attainment (Berkowitz, 1989)[8]. Although aggression frequently is directed toward the agent perceived to have provoked it, sometimes other features of the situation elicit restraint. Miller (1941)[47] proposed several constraining factors: (a) the provoking agent is unavailable (e.g., the provocateur has left the immediate environment), (b) the source of frustration is intangible (e.g., bad weather or a foul odor as in Konecni & Doob, 1972; Rotton, Barry, Frey, & Soler, 1978)[48][71], and (c) retaliation or punishment is feared from the provoking agent (e.g., the provocateur is one's boss or has other sources of power). When any of these constraining factors are present, direct aggression is often controlled (e.g., Bandura, 1973; Baron, 1971; Taylor, Schmutte, & Leonard, 1977)[2][4][68]. Instead, it is alleged to be redirected toward or displaced onto less powerful or more available targets, as described in our opening vignette. Baron and Bell (1975)[3] provide an empirical example that is based on the second of these restraining factors in that the source of a frustrating initial provocation was intangible. Thus, in the first stage of their study, the ambient temperature of a room was manipulated (i.e., hot and humid vs. normal) during a filler task. In the second stage, anger arousal was manipulated by a confederate who either insulted (or did not insult) the participant. In the final stage, the same confederate served as the learner in a modified teacher/learner paradigm in a new room and thus was available as a target of displaced or triggered displaced aggression (depending on whether insult was absent or present in the second stage of the experiment)[70]. The dependent variable was the duration and intensity of shock across 20 trials. In sum, there was a manipulation of the presence of an initial provocation (i.e., hot and humid vs. normal temperature) and a subsequent opportunity to aggress against a target who had or had not provided an act (insult or no insult) that by itself could function as a triggering provocation unrelated to the initial provocation.

Since, in this paper, I will also try to formulate the guilt aversion. Another way to look at it by introducing guilt aversion. In this game, the player if punishes the other player out of frustration, then guilt doesn't let him do that much harm. Guilt averse individuals experience a utility loss if they believe they let someone down (Ellingsen, Magnus Johannesson, Sigve Tjøtta, Gaute Torsvik, 2009)[56][13][50].

Most economic models assume that agents maximize their expected material payoffs. However, subjects in the lab exhibit persistent and significant deviations from this self- interested maximizing behavior. A reasonable explanation for this behavior is that players can be motivated not only by material (monetary) payos but also by what are sometimes referred to as psychological utilities[54]. These are related to preferences that are in some degree other regarding they take others into account. Traditional game theory does not provide enough tools to adequately describe many of these preferences: the traditional approach assumes that utilities only depend on the actions that are chosen by the players. By contrast, when players are emotional or motivated by reciprocity or social respect, their utilities may also directly depend on the beliefs.

Standard theories in economics generate predictions of market behavior by invoking two fundamental assumptions. First, agents are self-interested in that their utility function depends only on their own material payoffs. Second, market behavior is at equilibrium so that no individual agent can achieve a higher payoff by unilaterally deviating from the equilibrium. Recent advances in behavioral economics relax both assumptions by allowing agents, for example, to care about others' payoffs and to make mistakes (see Matthew Rabin 1998[57]; Colin F. Camerer[29], George Loewenstein[49], and Rabin 2004[68]; and Ho, Noah Lim, and Camerer 2006[51], for comprehensive reviews). This paper,"Peer-Induced Fairness in Games By Teck-Hua Ho and Xuanming Su[73] focuses on the self-interested assumption and investigates how social comparison may lead to fairness concerns between peers. A simple and powerful way to demonstrate that people are not purely self-interested is to study the so-called ultimatum game. In this game, a leader and a follower divide a fixed pie. The leader moves first and offers a division of the pie to the follower. The follower can accept or reject. If the follower accepts, the pie is distributed according to the proposal. If the follower rejects, both players earn nothing. When players care only about their own material payoffs, the subgame perfect equilibrium predicts that the leader should offer a small amount (e.g., a dime) to the follower and the follower would accept (since a dime is strictly preferred to nothing).

Taking into account that evolutionary pressure is driven by material payoffs (e.g., Buss 2016)[6], this result is consistent with the work of Sell, Cosmides, & Tooby (2009)[36], who argue that anger is the result of a process of natural selection for behaviors that resolve bargaining conflicts in favor of the anger-prone individual. We also formally develop the notion of a threat in order to provide a partial characterization of SE with anger: the presence of threats allows anger-prone followers to obtain more than in the material-payoffsequilibrium and give less to the leader, while their absence implies that equilibria with SA, ABB, or ABI are equivalent to the material payoffequilibrium.

In particular, a consumer may wish to spend some of those dollars in avoiding disappointment, an aspect of risk aversion that does not seem to be reflected by a utility function over dollar assets alone. This paper[45] does not suggest that people ought to make financial trade-offs to avoid disappointment, nor does it assert (though I believe it to be true) that people do so. As is the case with normative analyses, it merely indicates the behavior that is the logical result of such an objective. Although the implications of this analysis are, in a number of ways, consistent with behavior observed in laboratory experiments, it would be surprising to hear that subjects become sufficiently involved with their hypothetical choices to make psychological effects primary motivators of their selections. It may be that the psychological impacts of a decision are generated by the same thought process used in making a decision, namely that the value of an outcome is judged relative to various reference points such as status quo, foregone assets, and prior expectations. Disappointment, and related concepts such as regret, have important implications for the study of decision making under uncertainty. Although the axioms of von-Neumann and Morgenstern are the cornerstones of decision analysis, they cannot be expected to hold if preference has not been calculated over all attributes of interest to the decision maker. While it has taken a study of descriptive be-

havior to force recognition of the importance of psychological impacts to the decision maker, it is not our intent to revise the normative theory continually until it matches empirical evidence.
This is not to say that traditional game theory is not able to analyze the influence of feelings, emotions and social norms on the players behavior. Distribution-dependent preferences à la Fehr and Schmidt (1999)[44], for example, can be addressed by the traditional game theory. But when we deal with intention-based feelings, emotions and social norms, i.e. belief-dependent motivations, we need to turn to psychological game theory. This new framework focuses on strategic settings where at least one player has belief-dependent motivations or believes, with a certain probability, that one of his opponents has belief-dependent motivations. Nonetheless, it allows for every other kind of social preferences. In that sense, it can be interpreted as a generalization of the traditional game theory. In light of personal guilt aversion, whether an opportunity for communication exists or not is a key determinant of the feeling of guilt. Particularly, the existence of an opportunity for communication (or, more generally, an action that can raise another person's expectation) is a prerequisite for the feeling of guilt. However, in the literature, the guilt aversion hypothesis and alternative hypotheses have been experimentally compared only under settings where subjects have no opportunity for such communication. As a result, the validity of personal guilt aversion cannot be tested by any extant experiment. Motivated by this observation, we design and conduct a new experiment that can test the personal as well as original versions of the guilt aversion hypothesis.Specifically, we experimentally investigate a trust game with hidden action (Berg et al., 1995[7]; Charness and Dufwenberg,2006)[21]. In one of two treatments, this game is associated with the opportunity for pre-play communication while no such opportunity is provided in the other treatment. Additionally, as in Ellingsen et al. (2010)[39] and Reuben et al. (2009)[62], we let second movers (trustees) be informed about the beliefs of paired first movers (trustors), while the beliefs are elicited in such a way that it is incentive-compatible for first movers to report true beliefs.5Note that for the above game, both the original and personal versions of the guilt aversion hypothesis imply that in the with-communication treatment, the more the second mover believes that the first mover believes that the second mover takes the trustworthy action, the more often the second mover actually chooses the trustworthy action.In contrast to this prediction, we find the following evidence. The correlation between the elicited beliefs of first movers,which are the same as the second-order beliefs of second movers by the above design, and (trustful or trustworthy) behavioris almost zero and even slightly negative in the with- and without-communication treatments, respectively. In this sense,our results suggest that the role of guilt aversion may be smaller than what was previously believed. In this sense, we provide an additional case for the counterargument advanced by Vanberg (2008)[74] and Ellingsen et al. (2010[38]) to the guilt aversion hypothesis.

3 Preliminaries

In this section, they have defined the whole notations and definitions to be used for frustration, anger and blame. They have used a system of beliefs, i.e. first-order and second-order beliefs. The way I have extended he paper is to introduce the guilt sensitivity parameter and see how the results change due to that additional parameter.

Notations

The whole paper consists of two-stage game form which describes the rules and actions' of players. They are considering a simultaneous move game at each stage in the game. The following notations are considered:
a) $(a^t) = (a_i^t)_{i\epsilon I}$ is the set of action profiles for all players i
b) h - Histories at the root level and h $= \phi$ is the empty history (root); $h = (a^1, a^2)$ is the a history of length 2, which is terminal in the case of this paper. But, in general, we can have a game with n length. 'n' length can be given by $h = (a^1, a^2, a^3, ..., a^n)$ have action profiles of n stages.
c) H - Set of non-terminal histories
d) Z - Set of terminal histories (end notes)
e) $A_i(h)$ - Set of feasible actions of i given $h \epsilon H$. This set is a singleton if i is not active given h. A player can have n number of feasible actions to be played and the material payoffs will be attached to each level corresponding to each player and his action profile.
f) $I(h) = \{i\epsilon I : |A_i(h)| > 1\}$ is the set of active players given h. This denotes that the set of players must be always greater than 1. If set of active players is 0, then that level will not be played and it's not of any economic significance to the game. It's mentioned in the paper that in a game of perfect information, $I(h)$ is a single for each $h\epsilon H$. This means that in a perfect information game, each player is informed completely about the other player's actions that may have previously occured. So, each information set has to be singleton either only a single player is moving at that node and if more than one person are moving, then a dotted circle will be made around it showing a simultaneous move game. $I(h) \subseteq I$
$g) Set of feasible actions -A_i(h) = \times i\epsilon I A_i(h)$ is the set of action profiles for palyer i and $A_{-i}(h) = \times j \neq i A_j(h)$ is the set of all feasible actions for all players other than i. This includes active as well as non-active players in the game.
h) The material consequences of players' actions are determined by a profile of monetary payoff functions $(\pi_i : Z \to R)_{i\epsilon I}$
i) In the case of perfect information game, they have assumed that it leads to no relevant ties which means that if there are more than one terminal nodes, the player at that longest terminal node will get different material payoffs at different nodes. If relevant ties are there, then the player may get same payoff at different nodes and then, we have to introduce other factors to choose either of those or a positive probability needs to attach with both the outcomes and then, the results can be analyzed. Closure is used to further use the Weierstrass theorem to prove the maximum or minimum. For two stage games, both the player would have different material payoff in the first stage and different payoffs in the second stage to avoid any confusion which state to pick. There would be no chance moves in this game as assumed by them which means that there would be no nature or dummy player playing an economic importance by them.
j) π_i denotes the material-payoff for player-i
k) The standard precendece relation is defined: $\prec$ for histories in $H \cup Z$. The way it is

defined is $\forall h \epsilon H$, $i \epsilon I$, $a_i \epsilon A_i(h)$, it holds that $h \prec (h, a_i)$ and $(h, a_i) \prec (h, (a_i, a_{-i}))$ if there are other players than i who are active at h.

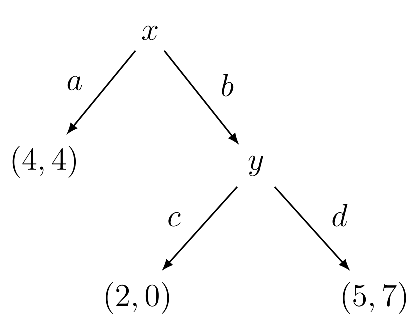
Game form

In this game form, they will introduce a dummy player c (with $c \epsilon I$). This player can choose a feasible action at random which won't play much economic significance. Now, the new set of players would be $I_c = I \cup c$. In this game, any move can be described ny the probability mass function $\sigma_c(.|h) \epsilon \Delta(A_c(h))$. The game described below will clear the notations as described in the previous section:

Beliefs, First-order and Second-order

In this paper, the author has talked about three different types of beliefs:
a) The player's plan, i.e. his own action profile which he has available at each step
b) belief about co-player's action, denoted by α
c) beliefs about co-players' belief, denoted by β

One thing to note about the beliefs is that these are conditional upon the histories of each active player. Player-i's beliefs can be dependent upon his own action and about other player's beliefs and actions, so it be described in the space of $Z \times \Delta_{-i}$. Any event which would take place for player-i would be $\subseteq Z \times \Delta_{-i}$. Since, behaviour would form a part of history and the co-players' beliefs, it can be decrobed in the set of $Y \times \Delta_{-i}$ where $Y \subseteq Z$. But, if we consider the space of beliefs, it will be $Z \times E\Delta_{-i}$ where $E\Delta_{-i} \subseteq \Delta_{-i}$
To determine the subjective value of the actions which are feasible for each player. We not only add history h to a players' belief but also the action profile chosen by the player (a_i). If a game of perfect information is taken in consideration, then $(h, a_i) \epsilon H \cup Z$. But, if a simultaneous move game, (h, a_i) won't be a history anymore. It'll not be known to the player what the co-player is moving. But player-i can choose an action which can't be reversed, he observes (h, a_i), and determine the material payoffs of each node for each player. H_i be the set of the histories which are non-terminal (standard) and by $Z(hi)$, the author means the terminal histories of h_i. One point to be noted is that if $h \prec h' \Rightarrow Z(h') \subseteq Z(h)$, with equality if no player is active at node h. Updated higher-order beliefs, beliefs of others, and plans of action may influence motivation, and another extension can be to capture dynamic psychological effects (such as sequential reciprocity, psychological forward induction, and regret) that were previously ruled out.



If $h = b$, then $Z(h) = ((b, c), (b, d, e), (b, d, f))$ and if there is another leg at d, then h' will occur, after h, Let's say that there is another leg at d as (e, f). $h' = e$, in this case, $Z(h') = ((b, d, e), (b, c))$. We can clearly see in this case that $Z(h') \subseteq Z(h)$.

First-order belief systems:

For every player-i, he hold belief about co-player's beliefs α_i which can be denoted by $\alpha_i(.|Z(h_i))\epsilon\Delta(Z(h_i))$ about the actions that will be taken in the whole game. According to the authors, the system of beliefs $\alpha_i = \alpha_i(.|Z(h_i))$ should satisfy two properties:
a) The rule of Bayesian probabilities should hold in this system wherever it's possible. If $h \prec h'_i$, then for every $Y \subseteq Z(h'_i)$

$$\alpha_i(Z(h'_i)|Z(h_i)) > 0 \Rightarrow \alpha_i(Y|Z(h'_i)) = \frac{\alpha_i(Y|Z(h_i))}{\alpha_i(Z(h'_i)|Z(h_i))}$$

If I try to open up the Bayes' Rule, then:

$$\alpha_i(Y|Z(h'_i)) = \frac{\alpha_i(Y \cap Z(h_i))}{\alpha_i(Z(h_i))} / \frac{\alpha_i(Z(h'_i) \cap Z(h_i))}{\alpha_i(Z(h_i)}$$

If $\alpha_i(Z(h_i) \neq 0$, then, it'd clearly observable by the Bayes' rule that Left Hand side will be equal to Right hand side. And, the game is formulated will prove that $\alpha_i(Z(h_i) \neq 0$ because a player-i's belief about the co-players' beliefs can not be 0. Since he knows that the other player has to play something if it's of economic significance and not a dummy player. So, this rule will always hold true. Some of the abbreviations used can be: $\forall h \epsilon H$, $a = (a_i, a_{-i})\epsilon A_i(h) \times A_{-i}(h)$

$(1)\alpha_i(a|h) = \alpha_i(Z(h, a)|Z(h))$
(2) $\alpha_{i,i}(a_i|h) = \sum_{a'_{-i}\epsilon A_{-i}(h)} \alpha_i(a_i, a'_i|h)$

The above equation will yield equation (1) automatically if it's summed to players' other than i. So, the player-i beliefs about the other players' all the actions will sum to 1. So, if the first-order beliefs are summed over all the action profile of co-player actions, that will sum to 1 given a history h.
$\alpha_{i,-i}(a_i|h) = \sum_{a'_i\epsilon A_i(h)} \alpha_i(a'_i, a_{-i}|h)$
The above equation states the player-i's beliefs about co-players' actions and other players' beliefs which will be summed upon the player-i action profile. These are just the definitions given by the authors, out of which some can be explained.

The above set of equations will imply a new definition as follows: $\alpha^{i,i}(a_i|h) = \alpha_i(Z(h, a_i)|Z(h))$. This will be given by the Bayes' rule that player-i's belief about the other players' belief and the terminal histories.
Those imply including (1) that $\alpha_i(a^1, a^2|\phi) = \alpha_i(a^2|a^1)\alpha_i(a^1|\phi)$. This is true since the a^2 is independent of a^1, ϕ is the empty history, so using Bayes' rule we can prove the previous

equality.
Player-i's beliefs about the actions taken at the same time by the co-players will be independent of player-i's action a_i: $\forall h \epsilon H$, $i \epsilon I$, $a_i \epsilon A_i(h)$, and $a_{-i} \epsilon A_{-i}(h)$,
$\alpha_{i,i}(a_(- i)|h) = \alpha_{i,-i}(a_{-i}|h, a_i)$
$\alpha_i(a_i, a_{-i}|h) = \alpha_{i,i}(a_i|h)\alpha_{i,-i}(a_{-i}|h)$

So, it can be observed that α_i is composed of two parts: what player-i believes about his own behavior for the whole game, and about the behavior of co-players'. Authors' aptly describe the plan of action of player-i as $\alpha_{i,i} \epsilon \times_{h \epsilon H} \Delta A_i(h)$ by the array of probability. If there are more than one player in the game (co-players), then $\alpha_{i,-i}$ correspond to the "correlated behavior strategy", where both the players will play according to their best response and plans. The point to note is that the players' plan won't describe his actual choice, but the actual action chosen in the path will be the actual choice.

First order belief of player-i will satisfy equation (1) and (2). The space of this belief will be represented by Δ_i^1. This can be checked easily that it will be a compact space because it's bounded by 0 and 1. Belief can't be less than 0 and it won't sum more than 1. Similarly, it holds for the co-players first-order beliefs and is represented by $\Delta_{-i}^1 = \times j \neq i \Delta_j^1$

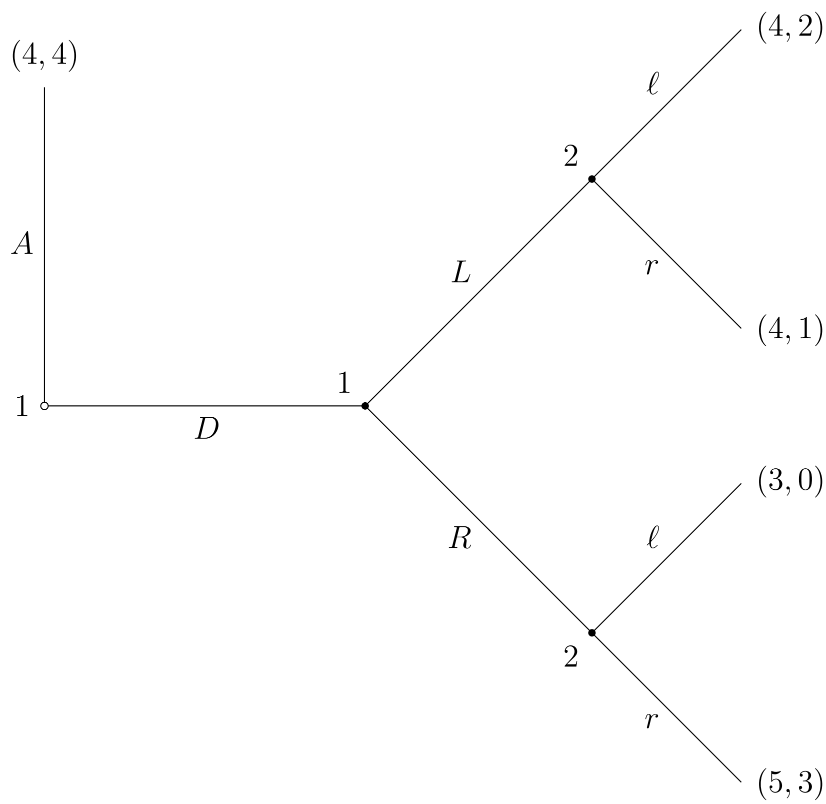


Figure: An extension game

Here we can define the notations well in this diagram, we can define the conditional probabilities, the way we were defining them earlier. I'll try to define the precedence relation in this tree-diagram also. The way to define the above equations can be:

$h = L$ and similarly, since,we know $h \prec h'$, so I'll take $h' = l$
Z is defined as the terminal notes with all the histories possible. So, I try to verify the above equations:

$Z(h) = ((A, D, L, l), (A, D, L, r))$
$Z(h') = ((A, D, L, l))$

We can clearly observe from the above equations that $Z(h') \subseteq Z(h)$. The intuition behind this is that if h is occurring before h', then all the terminal nodes of h will definitely include the terminal nodes of h', and these will be equal, i.e. $Z(h) = Z(h')$, when there is no other action at the h', that is if the player has no option in choosing h', basically it won't be of much economic significance because to end the game, the player must have to choose the action at h'. That will only happen if h' is the only feasible action and the game is ending or not, it doesn't matter. Because where the player has no choice in action can occur in the middle of the game even and h' can be defined there itself.

The only point to note is that according to the definition, h should precede h', i.e. it should occur before h'. Notation can be changed but the whole point of the definitions is clear.

Second-order belief systems:

This tells that any player not only hold beliefs about the paths, but also about the belief about the co-players.There are many extensive game forms such as finite or infinite which specify material payoffs for each player at the each end node (terminal nodes). These payoffs describe the material consequences of players' actions, not their preferences.The players' utilities will be introduced later in this paper. In this paper, the authors described it in the way that the co-players' beliefs are affected only by the first-order belief and values of actions. Second-order beliefs are defined by the system of conditional probability, $(\beta_i(.|h)_{h_i \epsilon H_i} \times_{h_i \epsilon H_i} \Delta(Z(h_i) \times \Delta^1_{-i})$. The properties which were satisfied in the first-order belief systems are also satisfied here as well for second-order,

If $h_i \prec h'_i$, then

$\beta_i(h'_i|h_i) > 0 \Rightarrow \beta_i(E|h'_i) = \frac{\beta_i(E|h_i)}{\beta_i(h'_i|h_i)}$ $\forall h \epsilon h_i, h'_i \epsilon H_i$, and every event $E \subseteq Z(h'_i) \times \Delta^1_{-i}$. Important point is note is that player-i's choice won't be influenced by co-players' beliefs and simultaneous choices. By the independence property, β_i will satisfy the following property:

$\forall h_i, h'_i \epsilon H_i$ and every event $E \subseteq Z(h_i) \times \Delta^1_{-i}$

(3) $\beta_i(h_i|h'_i) > 0 \Rightarrow \beta_i(E|h'_i) = \frac{\beta_i(E|h_i)}{\beta_i(h'_i|h_i)}$
(4) $\beta_i(Z(h,(a_i,a_{-i})) \times E_\Delta|(h,a_i)) = \beta_i(Z(h,(a'_i,a_{-i})) \times E_\Delta|(h,a'_i))$

The above equation (4) holds because of he independence axiom, if we try to open up the events and use Bayes' rule of $Pr(A|B,C,D,E) = \frac{Pr(A \cap B \cap C \cap D \cap E)}{Pr(B|C,D,E)Pr(C|D,E)Pr(D|E)Pr(E)}$
The space of Player-i's second order beliefs is given by Δ^2_{-i}.

An important thing to note in this case is that α_i (First-order belief of player-i) is implicit in second-order beliefs systems of β_i. The second-order belief system will satisfy equation (4) and (5). Since, any player will first form second-order beliefs and then, will move to first-order belief systems, so, it is rightly said by the authors that α_i is derived from β_i. Empty history is sometimes not written, but it's can be comprehended easily like, $\beta_i(E) = \beta_i(E|\phi)$ $\alpha_i = \alpha_i(a|\phi)$. So, it's just the notation written in this way.

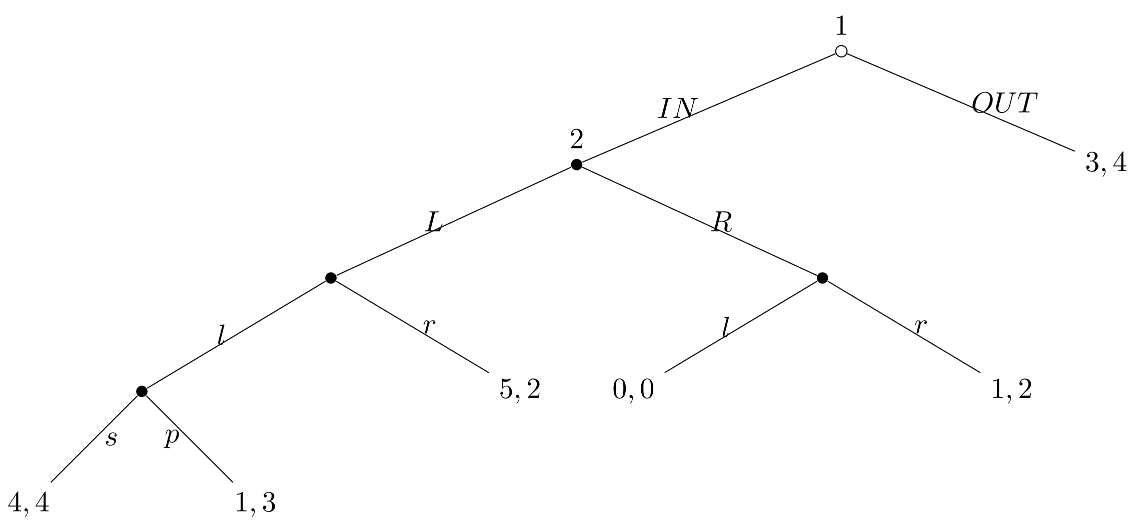


Figure: C:

Now, we can define the players and notations which we were defining early.

$H(Non-terminalNodes) = \{\phi, (IN, L, l)\}$
$Z(TerminalNodes) = \{(IN, L, l, s), (IN, L, l, p), (IN, L, r), (IN, R, l), (IN, R, r), OUT\}$
Active player at the root level:
$I(\phi) = \{1\}$ (only player-1 is active at the root initial)
$I(L, R) = \{2\}$ (active player at the node mentioned)
$A_1(\phi) = \{IN, OUT\}, A_2(IN) = \{L, R\}$ (feasible action sets for the players at that node either of the players)

Conditional Expectations:

The authors have defined a new function ψ_i which can measure any measurable function (real-valued). For example, it can take exmaple of co-players' second-order or first-order beliefs. The expected value of ψ_i conditional on any history $h_i \epsilon H_i$ can be given by $E[\psi_i|h_i, \beta_i]$. But, if ψ_i is only dependent on the actions, i.e. on the path of terminal nodes, then expected value function can be derived by α_i instead of β_i. In this case the expected values of the payoff can be given by the following functions derived by α_i:
$E(\pi_i|h;\alpha_i) = \sum_{z\epsilon Z(h)} \alpha_i(z|h)\pi_i(z)$

$E(\pi_i|(h, a_i);\alpha_i) = \sum_{z\epsilon Z(h,a_i)} \alpha_i(z|h, a_i)\pi_i(z)$

This will hold for $\forall h\epsilon H, a_i\epsilon A_i(h)$. In these equations, π_i is just the monetary payoff while α_i values are the probabilities attached to yield the expected value. $E(\pi_i|h;\alpha_i)$ is player-i's expectation payoff conditional on h given α_i. This also specify player-i's plan. $E(\pi_i|(h, a_i);\alpha_i)$ will be player-i's expected payoff given action profile a_i. If a_i is what payer-will choose at h, then $\alpha_{i,i}(a_i|h) = 1$, and the above two equations mentioned will yield the same payoff.

4 The frustration-aggression hypothesis, anger and Blame

Frustration

Frustration is a result of anger. Now, we need to find out a way of defining frustration which can also be tested in lab experiments. Basically, a function should be such that also applies to real life outcomes. If a function is introduced with games in it, then it's tested in Lab experiments in different places so that outcomes are not affected. It should be the case that if frustration is tested, then there should be no pre-conceived notions about the same. Because game should be conducted like a vacuum as if there is nothing in the mind of the player before. Only in this case, we can get unbiased results. But, in real life, it's difficult to conduct these type of experiments with people who have no notions and are totally unbiased with respect to the game form. Not only this, sometimes theoretical models are totally correct but it's difficult to find something that fits in real-life examples. We need models where we can test for the impact of human behaviour on economic significance. Here, in this paper, the authors tried to give a formal way of doing so by defining frustration in a mathematical way and the, defining other functions and forming a simple analysis with it. The way the authors define it is by assuming an extensive game where there will be terminal and non-terminal nodes as well. With each terminal node, there will be a material pay-off associated. Say, here we have three types of game-forms- Simple anger (where a player-j is angry because of the co-player's actions); anger with blaming behavior (anger is aroused because of the behaviour of the plaer that he thinks that the co-player moved intentionally

to get a higher pay-off but instead, end up getting a lower pay-off) and Anger with Blaming Intentions (in this form, the player will consider co-players' beliefs about his own actions and then, according to the maximisation rule, will decide whether he wants to punish him or not).If player-i feels let-down due to his own actions after punishing the co-player, then a new parameter of guilt-aversion is introduced. That depends on many psychological factors how the players are perceiving this game to play. The threat and guilt will differ from individual to individual. There area lot of techniques to find the equilibrium (simple or sequential), but these are based on some assumptions like players should behave rationally in all circumstances. There are many frustration definitions which are defined but a lot of them are critized. Believers of phsychological study sometimes, seem to differ their opinions from economics definition. So, a clear and proper way of defining the definition should be there so that it adds to the society and further research can be done on it.
Frustrated players blame and get angry with the co-players' deviated actions and out of that, they tend to punish the other player by their actions depending on their frustration parameter defined as follows:

$$F_i(h, \alpha_i) = [E(\pi_i; \alpha_i) - max_{a_i \epsilon A_i(h)} E(\pi_i|(h, a_i))]^+$$

Where the plus sign in the superscript indicates the max function, i.e. $x^+ = maxx, 0$, so the frustration can never be negative. The first function in frustration function is given as that's the expected value of the material payoff given his belief and the other part is what he believes could have been best for him given his belief and history h. Two points to note are:

i) Diminished expectation should exit for frustration to be positive, i.e. $E(\pi_i|h; \alpha_i < E(\pi_i; \alpha_i)$ (Necessary condition)
ii) It must not be possible for player-i to close the gap in the frustration function.

At $h = \phi$, Frustration must be 0 since expected and actual will match exactly since at the root level, no player has taken any action. This can be seen from the following equation even:

$$E(\pi_i; \alpha_i) = \sum_{a_i^1 \epsilon A_i(\phi)} \alpha_{i,i}(a_i^1|\phi) E(\pi_i|a_i^1; \alpha_i)) \leq max_(a_i^1 \epsilon A_i(\phi) E(\pi_i|a_i^1; \alpha_i))$$

This equation denotes that stage-1 player-i's action are summed over all feasible actions at history ϕ (root level) and $\alpha_{i,i}$ is the probability attached to find he expected pay-off. This will be equal to the second argument if $\alpha_{i,i} = 1$,i.e. the player knows which action he is going to take in particular.
In the paper, assumptions are made in such a way that frustration are felt at end notes and won't influence subsequent choices as the game will be over. This assumption is true in the sense that since authors are introducing two-stage game only, this assumption will always be fulfilled. In 2-stage game, all frustartion can be defined in only stage-2 actions', i.e. a^2,

$$F_i(a_1; \alpha_i) = [E(\pi_i; \alpha_i) - max_{a_i^2 \epsilon A_i(a^1)} E(\pi_i|(a^1, a_i^2); \alpha_i)]^+$$

Frustration of payer-i at stage-2 will be expected value of his monetary pay-off less maximised value of monetary pay-off of all the actions feasible in stage-2 given stage-1 actions and his belief about co-players' actions. And then, we have to take that value if positive otherwise 0.

Simple Anger

A frustrated player is always motivated to hurt others, but it makes sense that he will hurt the other player only if making him hurt is not costly. If giving punishment costs him a lot, and his own pay-off is reduced a lot, then he won't go for it, instead it would be better for him instead to not hurt and play as per the co-players' beliefs are. The authors will consider different versions of frustration, anger and blame. Though other methods and functions we can introduce different cognitive appraisals and versions of blame. Blame can be introduced in the max function or in the minimum function or any other corner-solution models. But, here the form is given as described by the above-mentioned function. Now, the utility function is described dependent on the first-order belief, action of the player-i and history of the game:

$$U_i(h, \alpha_i; \beta_i) = E(\pi_i|(h, a_i); \alpha_i) - \theta_i \sum_{j \neq i} B_{ij}(h; \beta_i) E(\pi_j|(h, a_i); \alpha_i)$$

Here, as we all know it's dependent on second-order beliefs and first-order beliefs and other things which were mentioned before. In this paper and literature defined previously, it is mandatory to show the players' information also at all the nodes (even if root level or not)where they are active or not. Thus, the information structure of player-i would be a partition H_i of the whole set H that contains, as a sub-collection, the standard information partition of $H \cup Z$. Thus, the representation here is such that B_{ij} function is representing the blame function, i.e. it represents how much of the frustration is blamed on the co-player by player-i. In this paper, they have assumed that if the frustration is positive, he will definitely hurt the other player. There is no reason to be guilt sensitive. But, I'll try to introduce the function of guilt aversion later in the paper.

$$B_{ij}(h; \beta_i) \leq F_i(h; \alpha_i)$$

Blame will always be less than or equal to the frustration depending on the parameter of anger and frustration. In this paper, it's denoted by θ_i.

There are some remarks given in this paper which are:
Remark 1: The decision utility which the player-i gets at the root level should coincide with the expected pay-off the player given his action and his firs-order beliefs:

$$U_i(\pi, a_i; \beta_i) = E(\pi_i|a_i; \alpha_i)$$

We need to observe that when player-i is the only active player playing denoted by $h = a^1$, he will obviously determine the terminal node by his action in stage-two of the game.

If this happens, then it can be clearly observed that expected value of the pay-off will be equal to the actual monetary pay-off of the player-i. And, the utility function can then be replaced by the without expectation instead of the expected value. The authors have assumed that the players' utilities are a common knowledge to all the players' present in the game. This paper models a game with anger and frustration which are used for analysis so that those can be traced. These are also tested by many other economists. All the definitions which come up and any theory is tested numerous times before getting published. In lab testing experiments, sometimes facial expressions change of players, which may lead to a signal to co-player that he is getting frustrated and but these things are not written in this paper and thus, testing is difficult with these hurdles.

Our analysis, **simple anger(SA)**, is that player-i's tendency to hurt all of the other players and will be proportional to player-i's frustration. In this whole paper, the authors will tend to equate blame and frustration as well. That means,

$B_{ij}(h;\beta_i) = F_i(h;\alpha_i)$

Simple anger utility function can be defined as the normal utility but now the blame function will be equal to the frustration function.

$U_i^{SA}(h, a_i; \alpha_i) = E(\pi_i|(h, a_i); \alpha_i) - \theta_i \sum_{j\neq i} F_i(h; \alpha_i)E(\pi_j|(h, a_i); \alpha_i)$

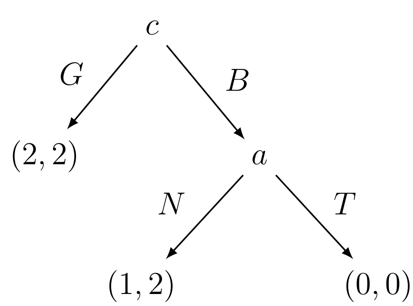


Figure: 1

To solve for simple anger in these games, and other games described later in this paper, we will use the above definitions,

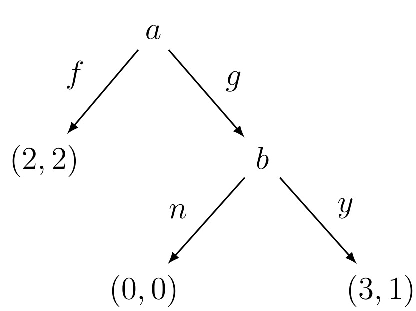


Figure: 2

In the above figure, Ann and Bob as depicted in the previous figure-2 will negotiate: Ann has two offers to make: {Fair, greedy}, where fair offer is at a terminal node, i.e. if selected, that will be automatically selected. If Ann chooses greedy offer where Bob has an option to either reject or accept the offer. Here, fair offer is represented by f while greedy offer is represented by g. His frustration following g is:

The second part of the frustration function would be given by:
$max_{a_b} 0.\alpha(n) + 1.\alpha(y) = 1.1 = 1$
To maximize the baove function, it would be best to take the value $\alpha(y) = 1$.

$F_b(g; \alpha_b) = [2.(1 - \alpha_b(g)) + \alpha_b(g).\alpha_b(y|g).1 - 1]^+$
Therefore, now we try to compare:

The first terms of $U_b^{SA}(g, n; \alpha_b) - U_b^{SA}(g, y; \alpha_b) = E(\pi_b|(g, n; \alpha_b)) - E(\pi_b|(g, y; \alpha_b)) - (\theta terms)$
$= 0 - 1 - (\theta terms)$

$U_b^{SA}(g, n; \alpha_b) - U_b^{SA}(g, y; \alpha_b) = -1 - \theta_b[F_b.E(\pi_a|(g, n; \alpha_b)) + \theta_b.E(\pi_a|(g, y; \alpha_b))]$

Now, we know by looking at the Figure-2 that $E(\pi_a|(g, n; \alpha_b) = 0$ and $E(\pi_a|(g, y; \alpha_b) = 1$

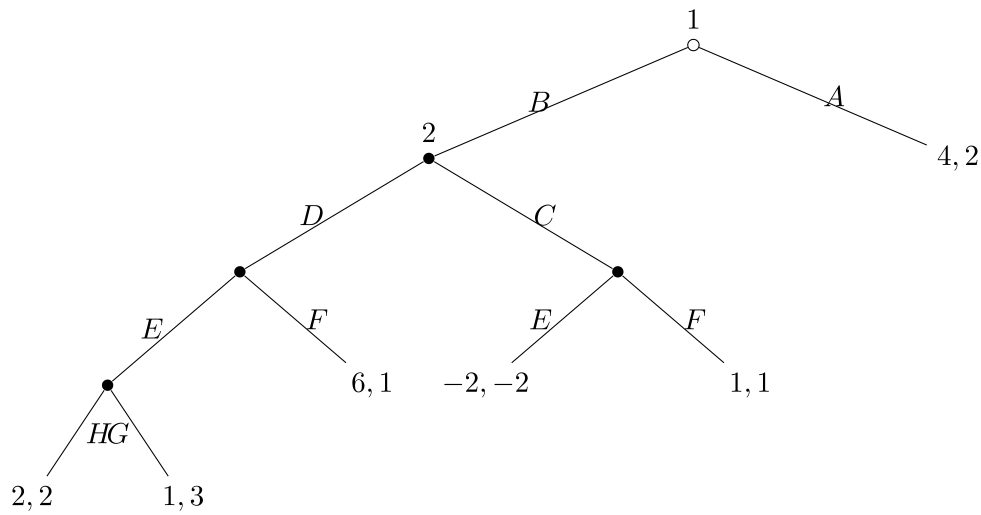
The above equation can be written in the form of:

$U_b^{SA}(g, n; \alpha_b) - U_b^{SA}(g, y; \alpha_b) = 3\theta_b([2.(1 - \alpha_b(g)) + \alpha_b(g).\alpha_b(y|g).1 - 1]^+ - 1)$

In the above equation if $\alpha_b(g) = 1$, then $U_b^{SA}(g, n; \alpha_b) < U_b^{SA}(g, y; \alpha_b)$

If this happens, then it would lead to no frustration which is not desirable in our case. In the above scenario, we have to note that Bob will be frustrated only if he expects $\alpha_b(g) < 1$, i.e. he has to expect happening with 'g' with less than 1 probability. The more 'g' he expects, the more frustrated he will be because he will expect Ann to give him a lesser pay-off than he would have got in getting a fair pay-off. One interesting point to is that the more Bob plans to reject, Bob will become less prone to reject the greedy offer once it will materialize. This model described above is also leading to the behavior which is non-consequential. If we hold the first-order and second-order beliefs constant, if Ann chooses 'g', then Bob will be more frustrated than he would have got in the case of 'f'. Because he didn't expect that to happen, so he gets more frustrated. This decreases the utility more in magnitude (material pay-off) than Ann would have chosen 'f'. This without any doubt will make the punishment more attractive to Bob and he tends to punish Ann a lot than in the normal case.

Bob rejects the offer and if he strictly prefers $n > y$. In this paper, the authors have made sure that Bob does not show that Ann choose greedy offer, then he will get frustrated and will punish if not costly. He is not giving any signal so that Ann does not get to know that Bob doesn't want her to choose 'g'. These things can also be considered and there are some categories like Reputational Models. Because of two-stage game, Bob won't be even able to influence the outcome further. Had Ann been choosing at stage-3, then frustration may have different results.



Now, in this diagram we will try to explain the notations of first-order and second-order beliefs. And whatever definitions we defined before, will try to justify the same.
Let $h_i = \phi$ and $Z(h_i) = \{A, BCE, BCF, BDF, BDEG, BDEH\}$
Let's define $h_i^{'} = \{BD\}$ and $Z(h_i^{'}) = \{BDF, BDEG, BDEH\}$
further we know the first-order beliefs of happening of the event $Z(h_i^{'})$
This means that where C is a large set:

(1) $\alpha_i(BDEG|Z(h_i) = \frac{1}{2}$

(2) $\alpha_i(BDEH|Z(h_i) = \frac{1}{3}$

(3) $\alpha_i(BDEF|Z(h_i) = \frac{1}{6}$
$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A \cap B|C)}{Pr(B \cap C|C)} = \frac{Pr(A \cap B|C).Pr(C)}{Pr(B \cap C|C).Pr(C)}$

$\alpha_i(Z(h_i^{'}|Z(h_i))) > 0$

$\alpha_i(A) = \frac{1}{3}$; $\alpha_i(BCE) = \frac{1}{6}$
$\alpha_i(BCF) = \frac{1}{6}$; $\alpha_i(BDF) = 0$
$\alpha_i(BDEH) = 0$; $\alpha_i(BDEG) = \frac{1}{3}$

$\alpha_i(BDEG|Z(h_i)) = \frac{\alpha_i((BDEG \cap Z(h_i))|Z(h_i))}{\alpha_i(Z(h_i^{'}|Z(h)))}$

These are four probabilities in $Z(h_i^{`})$ to happen; (BDEG, BDF, BDEH); considering $Z(h_i)$ i.e. the probabilities associated with the given terms, (BDF & BDEH) are occurring with 0 probability. This implies that, (BDEG) will occur with full probability. Because if we consider the event $Z(_i^{'})|Z(h_i)$, then given the latter event, if we consider the sub-part of it, then the elements in $h_i^{'}$ but not in $Z(h)$ would occur with 0 probability.
Let's assume $h = \phi$, $a = (a_i, a_{-i}) = (a_1, a_2, a_3)$
And that is also true: $\epsilon((AE, AF, BE, BF), (C, D), (G, H)))$
The first two elements of the above set belong to player-1 stage-1 actions, the next two elements will belong to player-1 stage- 2 game, the next element belongs to player-2 stage-1 game and the last two elements will belong to player-3 stage-1 game. So, all players have their feasible actions defined.
$Z(h) = \{A, BC, BDEG, BGEH, BDF\}$

$\alpha_{i,i}(a_i|h) = \sum_{a'_{-i}\epsilon A_{-i}(h))}(\alpha_i((a_i, a_{-i})|h))$

If only two players are considered,
$\alpha_{1,1}(a_1|h) = \sum_{a'_{-1}\epsilon A_{-1}(h))}(\alpha_1((a_1, a_{-1})|h))$

Let $a_1 = (A, B, C)$ and $a_{-1} = (D, E)$
$= \alpha_1(a_1, AD|\phi) + \alpha_1(a_1, AE|\phi) + \alpha_1(a_1, BD|\phi) + \alpha_1(a_1, BE|\phi) + \alpha_1(a_1, CD|\phi) + \alpha_1(a_1, CE|\phi)$
$= \alpha_{1,1}(a_1|\phi)$

$\alpha_{1,-1}(a_{-1}|h) = \alpha_1(a_{-1}, AD|\phi) + \alpha_1(a_{-1}, AE|\phi) + \alpha_1(a_{-1}, BD|\phi) + \alpha_1(a_{-1}, BE|\phi) + \alpha_1(a_{-1}, CD|\phi) + \alpha_1(a_{-1}, CE|\phi)$
This is for a particular action profile a_i.
$\alpha_i(a^2|a^1).\alpha_i(a^1|\phi) = \frac{\alpha_1(a^2 \cap a^1)}{\alpha_1(a^1)}.\frac{\alpha_1(a^1 \cap \phi)}{\alpha_1(\phi)}$
Since ϕ is a larger set, it will occur at the first node and intersection with it will be ϕ only.
$\alpha_i(a^1, a^2|\phi) = \alpha_1(a^1 \cap a^2|\phi)$

$\alpha_{i,i}$ is the players' own belief about his own action while $\alpha_{i,-i}$ is his own belief about other players' actions.
In the previous equation, right hand side indicates that given the game is played, player-'i"s belief regarding stage-1 (a_1) to be played. By Bayesian rule, it is multiplied by player-i's belief regarding stage-2 action profile given stage-1 is played. Left hand side indicates that there is some active player at ϕ (largest set), player-i's belief regarding an action profile of stage-2 and stage-1 game being played.
Δ_i^1 is the compact set becuase this is the set of the first-order beliefs and it is compact since $0 \leq \alpha_i \leq 1$ and union of compact sets(closed and bounded) will also be a union set. For a particular individual-i, this space is defined over terminal finite of nodes.

$\alpha_{i,i}(a_i|h).\alpha_{i,-i}(a_{-i}|h) = \alpha_i(Z(h, a_i)|Z(h)).\alpha_{i,-i}(a_{-i|h;a_i})$

The first term on the right hand side is player-i forming belief about his own action and the second term is player-i forming belief about other's behavior given his own action, a_i. $\alpha_{i,i}$ is his own belief own action space and $\times_{h\epsilon H}$ represents for all histories (Non-terminal) and $\delta(A_i(h))$ is the space of action profile of player-i.

$\alpha_1 = (\alpha_1(.|Z(h_1)))_{h_1 \epsilon H_1}$ for all histories of player-1 that belongs to H_1

$\sum \alpha_1 = 1$ Each players' belief sum to 1
$\sum_{i=1}^{n} \alpha_i = n$ (If n players, total sum of beliefs would be equal to n)

Second-order beliefs are not influenced by by player-i's own action and even first-order beliefs are not altered. Each player while taking action consider other player's beliefs and his state of being will depend on other player's action.
In second-order belief systems, $(Z(h_i) \times \Delta^1_{-i})$ is the space of terminal nodes of histories of player-i with the beliefs at stage-1 about co-players.
It is given that:
$\alpha_i(Y|h_i) = \beta_i(Y \times \Delta^1_{-i}|h_i)$
If we sum over this space Δ^1_{-i}, we get α_i.

Anger from Blaming behaviour (ABB)

Action profiles can be taken depending on the player-i's intention to blame others. When any frustrated player-i tend to blame their co-players for their behavior and the actions taken by them, he critically examines the co-player's actions in action-1 (initial stage) without considering the other person's first-order beliefs of other players or their intentions. The way authors have defined the blame function is the following:

$$\mathrm{B}_{ij}(a^1;\alpha_i) = \begin{cases} 0, \forall j \notin I(\phi) \\ F_i(a^1;\alpha_i), \forall j \epsilon I(\phi) \end{cases}$$

The function can be interpreted as: if the player is active in the initial root or the history of the game, then the blame function can be defined by the frustration function defined previously. And is the player is not active in the initial root of the game, then player-i puts no blame on it. That is, then the blame function would be 0 in that case. It is continuous can be observed that there are only two options that either the player-j would be active or not in the root level of the game.
If active, then the frustration function is defined with values taking from 0 to the upper bound as defined previously. If not, then blame is 0, since there is no reason to blame him since he is not active.
The above equation is a complete specification of the i-j blame function. But this is complete only if there is a first mover, but with two or more first movers, the blame function would be 0 only and the lower part of the blame function would become irrelevant. So, the upper portion is a necessary condition for it be be defined and continuous.
Aan importnat point to note is that if player-j is the only active player at a stage, then his co-player, i.e. player-i will put the blame fully onto him. Now, the utility function from ABB would be given by:

$U_i^{ABB}(h, a_i; \alpha_i) = E(\pi_i|(h, a_i); \alpha_i) - \theta_i \sum_{j \neq i} B_{ij}(h; \alpha_i)) E(\pi_j|(h, a_i; \alpha_i))$

This function describes the ABB utility function with a slight change than simple anger in the sense that it has a different blame function, depending on the behavior of the other person, he would blame his co-player otherwise not.
Here, a refers to the andyman who uses the hammer and the other player is Bob who is doing an apprentice, under him. Andy on a bad day, can hit his thumb with the hammer and take it out on Bob.If G is of the probability $(1 - \epsilon)$ and B of ϵ. Important assumption to make here is that $\alpha_a(B) = \epsilon < \frac{1}{2}$ (mainly to assume frustration to be positive and defined, will see in detail later why need this assumption)
Frustration function of a would be given by:

$F_a(B; \alpha_a) = (1 - \epsilon).2 + \epsilon \alpha_a(N|B).1 - 1$
$= 2 - 2\epsilon + \epsilon \alpha_a(N|B).1 - 1$
$= 1 - 2\epsilon + \epsilon \alpha_a(N|B).1 - 1$

We can see that the first order belief is either 0 or positive, so frustration to be positive , we need $\epsilon < \frac{1}{2}$. With SA and with θ_a sufficiently high, on a bad day Andy chooses T in a fit of displaced aggression. But, since Bob is passive, with ABB Andy chooses N regardless of θ_a.

Could-have-been blame

In this portion, player-i would think what he could have got in any other action profile, had the co-player chosen differently. Player-i is already frustrated, nw he try to assess the possibilities of what different outcomes had he got when all the co-players would behaved differently, i.e., $\forall j$. In expectation, that value can be written as:

$max_{a_j^{'} \epsilon A_j(\phi)} E(\pi_i|(a_{-j}^1, a_j^{'}; \alpha_i))$ If this could-have-been payoff is more than what i currently expects, then i blames j, up to i's frustration, i.e., there would a bound on it by the frustration function defining which function is higher. This will even satisfy that if player-j is not the active player, then he would not be blamed.

Bij $(\mathrm{a}_1; \alpha_i) = min \left[[max_{a_j^{'} \epsilon A_j(\phi)} E(\pi_i|(a_{-j}^1; \alpha_i) - E(\pi_i|a^1; \alpha_i)]^+, F_i(a_1; \alpha_i) \right]$

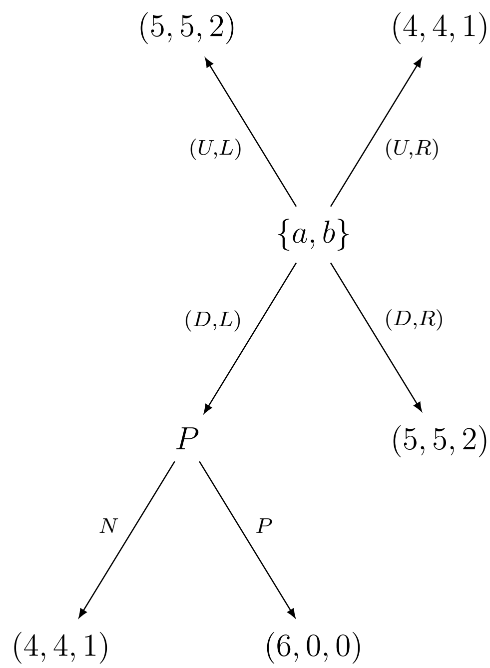


Figure: D

In the above figure, if we consider Penny at $a^1 = (D, L)$, i.e. in the first-stage game, let's rule out all other possibilities and focus on only (D, L) where P has to choose between N, P. If we maximise Penny's payoff w.r.t. both Ann and Bob, then $X = max_{a'_j \epsilon A_j(\phi)} E(\pi_i|(a^1_{-j}; \alpha_i)$

$X_p = max_{a_b \epsilon A_(\phi)}(E(\pi_b|(a^1_b), p); \alpha_p)$
Depending on what other players'(ann/bob) would choose, her expected payoff would be maximized at (D, L). One thing to note is that either we maximise wrt to ann or bob, this would yield the same result.
$X_p = 2(\alpha_{U,L}) + 1(\alpha_{U,R}) + 2(\alpha_{D,R})$
To maximise this expected payoff, if the second outcome i.e. (U, R) is played, then it would result in less expected payoff, so the second term vanishes and it will be 0.
$X_p = 2(\alpha_{U,L} + \alpha_{D,R}) = 2.1 = 2$
The term inside the brackets about the first-order beliefs would be equal to 1, since the player would assume that he will play in the game. So, that's how $E(\pi_p; \alpha_p) \geq 2$.

$E(\pi_p|(D, L); \alpha_p) = 1(\alpha(N|(D, L)) + 0 \leq 1$
$F_i(a^1; \alpha_i) = [E(\pi_p; \alpha_p) - 1]^+$
Now, we can clearly observe that the blame function on ann or bob will be identical, which can be represented by:

$B_{pa}((D, L); \alpha_p) = B_{pb}((D, L); \alpha_p) = min[[2 - E(\pi_p|(D, L); \alpha_p)^+, [E(\pi_p; \alpha_p) - 1]^+]] = [E(\pi_p; \alpha_p) - 1]^+$

This is happening because the first expected term in the brackets will always be less than

or equal to 1 (shown above), while the second term in the minimum expression would be always greater than or equal to 0, so to take the minimum expression, that would be the second term.
The above equation shows that bob and ann are fully blamed by penny for her frustration and if given an opportunity, she will take it out, by choosing the outcome which harms both of them.

Blaming unexpected deviations

When frustrated after a^1, i assesses, for each j (co-player), how much he would have obtained had j behaved as expected:

$$\sum_{a'_j \epsilon A_j(\phi)} E(\pi_i|(a^1_{-j}, a'_j; \alpha_i)$$

where $\alpha_{ij}(a'_j)$ is the marginal probability of a'_j according to the first-order belief of player-i. The blame function would be defined with this slight modification of submission instead of a maximum function and rest of the things would be similar as before.
Giving an example to define the whole function:
If player-j is not active in the first stage, then we can observe that $B_{ij} = 0$, we defined this even in the previous parts and this result is consistent with that part.

$$Bij(a_1; \alpha_i) = min\left[[E(\pi_i|(a^1; \alpha_i) - E(\pi_i|a^1; \alpha_i)]^+, F_i(a_1; \alpha_i)\right]$$
$B_{ij} = 0$
since j cannot have deviated, he cannot be blamed. If, instead, only j is active in the first stage, then the whole whole blame will be put onto him and since, no other player is active, the player-i will be frustrated and take the full blame onto player-j. This means, mathematically, that

$$B_{ij}(a^1; \alpha_i) = F_i(a^1; \alpha_i)$$

If player-j's action in stage-1 is what payer-j expected to do, then the marginal probability $\alpha_{ij} = 1$, then

$$B_{ij} = min\left[[E(\pi_i|(a^1; \alpha_i) - E(\pi_i|a^1; \alpha_i)]^+, F_i(a_1; \alpha_i)\right] = 0$$

This basically means the expectation of payer-i matches exactly and that's why blame is nil. That is, j did not deviate from what i expected and is not blamed by i, marking a contrast to could have been" blame (the previous one).
If we consider this type of function in the earlier game, then again Penny is fully certain of the outcome (U, L), then $\alpha(D, L) = 1$, so expected payoff of penny would be 2. The frustration function would be same as before:

$F_p((D, L); \alpha_p) = [E(\pi_p; \alpha_p) - 1]^+ = 1$
Now we know that ann has deviated from U to D, ann will be blamed instead of Penny. The summed term if calculated on ann's actions, then it would be equal be 2. The expected pay-off of Penny would be 2, i.e. $\pi_p(U, L) = 2$

Penny will blame as much as her frustration, that is,
$B_{pa}((D, L); \alpha_p) = min[2 - E(\pi_p|a^1; \alpha_p), 1] = 1$
Penny does not blame Bob, who played L as expected. To see this, note that after (D; L) Penny assesses how much she would have obtained had Bob behaved as expected, so she would also calculate the blame on Bob:

$$B_{pb}((D, L); \alpha_p) = min[(E(\pi_p|(D, L); \alpha_p) - E(\pi_p(D, L); \alpha_p))^+, 1] = 0$$

This is totally opposite to the case where penny fully blames bob but now, the blame is 0, since ann deviated and bob plays as expected.
In two-player games with a single leader and a single follower (two-stage games) ABB and SA (previously described forms) are behaviorally equivalent that is both are consistent with the previous assumptions. However, in games with more than two followers, with an inactive player in the second stage, or with chance moves, SA and ABB give varying predictions about behavior of players in both the games' forms.

Anger from Blaming Intentions

In this type of game, the player-i will now include the second-order beliefs, i.e. the player-i will now makes expectations of co-players' beliefs. The player will now think if the co-player intended to give him a lower pay-ff. He will now have some belief about their beliefs. One point to note is that this will depend on what player-j has belief about player-i's action, i.e. α_j, which will include his plan. His plan will include the expectation of his payoff given his action and his belief and player-i's belief about his action. So, basically, he will put himself in player-i's shoes and make the same computation as player-i did. That also depends on how much player-i will blame player-j for his payoff and expectations.
Getting in the shoes of player-i, player-j will maximize his expected pay-off initially:

$$max_{a_j^1 \epsilon A_{-j}(\phi)} \sum_{a_{-j}^1 \epsilon A_{-j}(\phi)} \alpha_{j,-j}(a_{-j}^1) E(\pi_i|(a_j^1, a_{-j}^1); \alpha_j)$$

Here, $\alpha_{j,-j}$ is the probability attached to the material pay-off associated with each node and this will be maximized by multiplying it wrt to the probability. This will be summed over all the actions feasible for player-j starting from the root level. This will then be maximized over the action profiles for player-j in stage-1.
A point to note with an important inequality is that:

$$max_{a_j^1 \epsilon A_{-j}(\phi)} \sum_{a_{-j}^1 \epsilon A_{-j}(\phi)} \alpha_{j,-j}(a_{-j}^1) E(\pi_i|(a_j^1, a_{-j}^1); \alpha_j)$$

$\geq \sum_{a^1 \epsilon A(\phi) \alpha_j} \alpha_j(a^1) E(\pi_i | a^1; \alpha_j) = E(\pi_i | \alpha_j)$

The first-order belief of player-'j' will be kept fixed under maximization. Which is meant that player-j believes that what he could achieve and take his action in the whole game (stage-1 and stage -2 combined). He can control his actions in stage-1, since he will taking actions according to his maximized expected pay-off but for the second stage, he can only predicts his actions, because the other player will also be taking actions which he can only guess according to the expected pay-off rule. So, stage-2 actions are not in his hand.

$$B_{ij}(a^1; \beta_i) = min \left\{ E[max_{a_j^1} \sum_{a_{-j}^1} \alpha_{j,-j}(a_{-j}^1) E(\pi_i | (a_j^1, a_{-j}^1; \alpha_j) - E(\phi_i; \alpha_j) | a_1; \beta_i], F_i(a^1; \alpha_i) \right\}$$

Further, the authors assume that player-i will blame player-j according to the difference between the maximised expected value of material pay-off of player-i given the first-order beliefs of player-i and the other players as well. This function will also be non-negative always, since frustration function is defined in the sense that wither it will be 0 or a positive number. And even the first term in the expression would be either 0 or positive, so minimum of those terms terms will be non-negative. But, to maintain consistency with the previous results, then we have to use a minimum function with the frustration value in it. If expected value of material pay-off of player-i summed over all feasible actions of other players in stage-1 is equal to the material pay-off of player-i, then Blame function would be capped by frustration function, otherwise the other term.

We know in any function, α_i is derived from β_i, since in any game form, second-order beliefs will be formed first and then, will move to first-order beliefs. No, the utility function described in this ABI form at stage-1 history would be given by the following (Blame function is described as previously):

$$U_i^{ABI}(h, a_i; \beta_i) = E(\pi_i | (h, a_i); \alpha_j) - \theta_i (\sum_{j \neq i} B_{ij}(h; \beta_i) E(\pi_j | (h, a_i); \alpha_i))$$

If we try to anlyse the same in Figure-2, as we discussed before too, the maximized pay-off that Ann can expect to give Bob is 2, independent of her first-order beliefs. Suppose, in the second-stage, bob observes that ann is taking 'g' in the hope of getting 3 as her pay-off instead of 2. But, this is gonna cost Bob a lower pay-off of 1, so Bob know this before and he might think to punish Ann by choosing n to give her a lower pay-off than expected.

With ABI in place, let's assume that Ann is planning to choose 'g' with probability $p < 1$, since Bob thinks that Ann is going to choose g with certainty, this means Bob beliefs about Ann choosing g is 1, i.e. $\beta_b(\alpha_a(y|g) = p|g) = 1$. Also, Bob is certain after g that Ann expected him to accept with probability q:

$\beta_b(\alpha_a(y|g) = q|g) = 1$. This means that Ann expected Bob that he will accept with probability q. Finally, suppose Bob initially expected to get the fair offer $(\alpha_b(f)) = 1$, this means that fair offer is chosen with the full probability.

The frustration function in stage-1 given the first-order beliefs of Bob defined as before can be computed:

$F_b(a^1; \alpha_b) = 2 - 1 = 1$

By summing over Ann's feasible actions and taking the probabilities as p for g and q for y, then the blame function can be found as:

$B_{pa}(g;\beta_b) = min\{2 - [2(1-p) + qp], 1\} = min\{p(2-q), 1\}$

If $p(2-q) < 1$, implying $(2p - pq) < 1$, then $B_{pa}(g;\beta_b) = \{p(2-q)\}$,
If q high enough or p is low enough, Bob does not blame all frustration on Ann. That means that Bob doesn't expect g to happen with high probability or expect f (probability q) to happen with high probability, then in both the cases, he is satisfied with Ann and won't put the blame on Ann. She gets some credit for initial intention to choose f with probability $(1-p) > 0$, and the credit depends on choosing qreedy offer, i.e. g (q).

Guilt-aversion

In this portion, I've tried to introduce the guilt-aversion function which means that if any player punishes his co-player, then he may feel guilty due to punishing him. So, now depending on the sensitiveness of the guilt, the player would punish accordingly.
The guilt and frustration moves in opposite directions because guilt aversion makes a player not to punish whereas frustration makes a player to punish. The basic rule would be define it in the simple way as:
G(Guilt aversion) = Expected Utility - a(The step he took to make the other person get less payoff)
where a will be some parameter for the degree of guilt aversion.
As can be clearly observable that Guilt will be defined only if Frustration is positive, since only if he is frustrated. If frustrated, he will tend to punish the other player and then, guilt-aversion might set in. If frustration is 0, then he has no incentive to punish, then Guilt is 0.
Let's assume G_i to be the guilt-aversion function of player-i and η_i is the guilt sensitive parameter:

$$G_i(h;\alpha_i) = \begin{cases} \eta_i(\pi_i(h|a_i) - E(\pi_i|a_i, a_{-i};\alpha_i)), F_i > 0 \\ 0, F_i = 0 \end{cases}$$

If the other player deviates, then frustration might come in and then, guilt might take place, then to punish, frustration would be positive ($F_i = 0$). But, if Guilty > 0, then this implies that the player-i feels guilty and might not punish the other player. If frustration $= 0$, then the other player has moved rationally and there is no incentive for player-i to feel frustrated or guilty.
In that case, $F_i(a_i, \alpha_i) = G_i(h;\alpha_i) = 0$

$$U_i^{SA} = E(\pi_i|(h, a_i);\alpha_i) - \theta_i \sum_{j\neq i} F_i(h;\alpha_i) E(\pi_j|(h, a_i);\alpha_i) + G_i(h;\alpha_i)$$

$$F_b(g;\alpha_b) = [(1 - \alpha_b(g)).2 + \alpha_b(g)(\alpha_b(y|g)).1 - 1]^+$$

$$G_b(g;\alpha_b) = \eta_b(1 - (\alpha_b(f).2 + \alpha_b(y|g).1))^+$$

Depending on η_b, guilt sensitivity will be measured. By plugging in the values of guilt-

aversion and frustration in simple utility, then utility function will be derived and can be used for further analysis. Frustration and guilt-aversion will act in opposite directions due to change in signs of both the equations since $\eta_i \epsilon [0, 1]$.
One important to point is of the extreme conditions if $\eta_b = 1 \& \theta_b = 1$:
In this case, (Frustration - Guilt-aversion function) = $(F_b - G_b) = 0$
It means that he is fully frustrated and puts the whole blame on Ann but he is also fully guilt-averse and then, in that case, due to the reaction in opposite directions, he won't punish Ann at all and both will cancel out each other.

5 Conclusion

In this paper, we have tried to formulate the effects of psychology and human behavior. Combining behavior and economics is a difficult task and requires in-depth understanding of the concepts. Giving a function and defining frustration or blame for analysis in economics, is not always applicable to each individual. But, it's not also possible to change the frustration function for each individual. Though, some parameters can be defined which varies for each individual separately. But still, they can't be exactly consistent with the way the individual formulates it. These papers can give an initial way of formulating anger, frustration and blame and how these can have an economic significance. Experiments and lab formulations are done to test these functions in real-life. Let's consider a new two-stage form, where the players have an only motive to maximise their material pay-offs. This is the common interest of both the individuals. Player-A has two options to choose from: {Left, Right}. If Player-A chooses {Left}, then the game will end at that node itself, and the payoff derived to Players- {A,B} respectively would be {3,3}. If the player-A chooses {Right}, then Player-B will get to choose {Up, Down}. If player-B would choose the former one, i.e. {Up}, then the material payoff for both would be {1,1} and in the latter case, the pay-off would be {5,2}. If Player-A is selfish and would choose instead Right to get a pay-off of {5,2}. But, upon observing this, Player-B may choose to punish the player and if θ is high enough, then he may actually choose the pay-off which gives less to both the players than the other one. With the games defined in this way, i.e. if we include simple anger or anger with blaming behavior, the θ will be defined and accordingly, actions will be chosen.
We will now take a binary gamble (lottery) with probability y where Charlie would win p amount which would be greater than 0 or otherwise, he would get 0. We'll assume that there is no cost to buying this (e.g. free lunches). Now, to find the expected value of this would be by finding the integration of the problem that would yield the answer as $p.y + 0.(1 - y) = p.y$, this shows that there are strong implications in whatever way the frustration come out. But, in this case, what if the p value is too high, there are high changes of an unexpected failure which will lead to frustration in this case, which will lead to all the implications which are given in this paper. But, there is a really less loss which the individual don't think about and then, bear the consequences of losing out, vetting out anger, frustration and blame.
In this paper, I've tried to develop a general theory of guilt aversion and show some simple results of how this can be taken out in the form of a game. I hope whatever I've tried to

formulate in this paper will be of economic significance and add some contributions. It seems obvious that all psychological effects lead to economic impact and guilt-aversion is one of them. These also seem to contribute towards the public goods and game theory as well as psychological concepts. Few other theories exist in the literature which are worth reading to.These theories of psychological game theory for studying many diverse kinds of motivation and other concepts are useful and well written for the framework's potential to analyze other phenomena in the literature also such as regret, shame, blame, guilt, disappointment, anger, excitement, happiness and joy.

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